Practice Numerical

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(1) The diameter of a wire measured in an experiment was 0.022 cm., 0.023 cm, 0.026 cm, 0.025 cm, 0.024 cm. and 0.025 cm. find →

(i) the mean value of diameter.

Sol^{n} :→ ~~$T_1 = 0.022$ cm., T_1~~

$L_1 = 0.022$ cm. $L_2 = 0.023$ cm, $L_3 = 0.026$ cm, $L_4 = 0.025$ cm., $L_5 = 0.024$ cm. , $L_6 = 0.025$

∴ The mean value → ~~$m =$~~ $L_m = \frac{0.022+0.023+0.026+0.025+0.024}{5}$

$$L_m = \frac{0.022+0.023+0.026+0.025+0.024+0.025}{6}$$

$$= \frac{0.145}{6}$$

$$= 0.2416 \ldots$$

$$\approx 0.242.$$

(ii) absolute error in a each measurement

$\Delta L_1 = L_m - L_1 = 0.242 - 0.022 = 0.220$ cm.

$\Delta L_2 = L_m - L_2 = 0.242 - 0.023 = 0.219$ cm

$\Delta L_3 = L_m - L_3 = 0.242 - 0.026 = 0.216$ cm.

$\Delta L_4 = L_m - L_4 = 0.242 - 0.025 = 0.217$ cm.

$\Delta L_5 = L_m - L_5 = 0.242 - 0.024 = 0.218$ cm.

$\Delta L_6 = L_m - L_6 = 0.242 - 0.025 = 0.217$ cm.

∴ Mean absolute error $= \bar{\Delta L}$

$$= \frac{|\Delta L_1| + \ldots + |\Delta L_6|}{6}$$

$$= \frac{|0.2201 + 0.219| + |0.216| + |0.2171 + |0.2181 + |0.2191}{6}$$

$$= \frac{1.302}{6}$$

$$= 0.217 \text{ cm}$$

(iii) percentage error $= \frac{\Delta \bar{R}}{R_m} \times 100\%$

$$= \frac{\Delta \bar{L}}{L_m} \times 100\%$$

$$= \frac{0.217}{0.242} \times 100\%$$

$$= 0.896 \times 100\%$$

$$= 89.6\%$$

— x — x — x —

(2) Refractive index of a flint glass (μ) was measured in an experiment and was found to be 1.655, 1.667, 1.666, 1.659, 1.669 and 1.654. Find,

(i) the mean value of μ

$\mu_1 = 1.655$

$\mu_2 = 1.667$

$\mu_3 = 1.666$

$\mu_4 = 1.659$

$\mu_5 = 1.669$

$\mu_6 = 1.654$

The mean value of $\mu_m =$

$$= \frac{1.655 + 1.667 + 1.666 + 1.659 + 1.669 + 1.654}{6}$$

$$= \frac{9.97}{6} = 1.6616$$

$$= 1.662$$

(ii) ∴ absolute error in a each measurement —

$\Delta \mu_1 = \mu_m - \mu_1 = 1.662 - 1.655 = 0.007$.

$\Delta \mu_2 = \mu_m - \mu_2 = 1.662 - 1.667 = -0.005$

$\Delta \mu_3 = \mu_m - \mu_3 = 1.662 - 1.666 = -0.004$

$\Delta \mu_4 = \mu_m - \mu_4 = 1.662 - 1.659 = 0.003$

$\Delta \mu_5 = \mu_m - \mu_5 = 1.662 - 1.669 = -0.007$

$\Delta \mu_6 = \mu_m - \mu_6 = 1.662 - 1.654 = 0.008$.

(ii) mean absolute error →

$$\Delta \bar{\mu}_m = \frac{|\Delta \mu_1| + \cdots + |\Delta \mu_6|}{6}$$

$$= \frac{|0.007| + |-0.005| + |-0.004| + |0.003| + |0.007| + |0.008|}{6}$$

$$= \frac{0.034}{6}$$

$$= 0.0056$$

$$= 0.006.$$

(iii) Relative error

$$RE = \frac{\Delta \bar{\mu}}{\mu_m}$$

$$= \frac{0.006}{1.662} = 0.0036.$$

$$= 0.004.$$

(iv) Percentage error $= \frac{\Delta \bar{\mu}}{\mu_m} \times 100\%$.

$= 0.004 \times 100\%$

$= 0.4\%$.

(3) While determining the time period of so oscillation of a simple pendulum, of the readings from various measurments are 1·73 s, 1·62 s, 1·52 s, 1·45 s and 1·83 s. Calculate the values of →

(I) mean value of time period →

$$T_m = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$$

$$= \frac{1.73 + 1.62 + 1.52 + 1.45 + 1.83}{5}$$

$$= \frac{8.15}{5} = 1.63 \text{ s.}$$

(II) absolute error

$\Delta T_1 = T_m - T_1 = 1.63 - 1.73 = -0.10$ s.

$\Delta T_2 = T_m - T_2 = 1.63 - 1.62 = 0.01$ s.

$\Delta T_3 = T_m - T_3 = 1.63 - 1.52 = 0.11$ s.

$\Delta T_4 = T_m - T_4 = 1.63 - 1.45 = 0.18$ s

$\Delta T_5 = T_m - T_5 = 1.63 - 1.83 = -0.20$ s.

(III) mean absolute error.

$$\Delta\bar{T} = \frac{|\Delta T_1| + \cdots + |\Delta T_5|}{5}$$

$$= \frac{|-0.10| + |0.01| + |0.11| + |0.18| + |0.20|}{5}$$

$$= \frac{0.6}{5}$$

$$= 0.12 \text{ s.}$$

(IV) $RE = \frac{\Delta\bar{T}}{\Delta T_m} = \frac{0.12}{1.63} = 0.073$

≈ 0.07 s.

(4) While determining the density of a sea water, the readings from various measurments ~~from~~ are ~~various~~ 1.03 g/cc, 1.12 g/cc, 0.92 g/cc, 1.05 g/cc and 1.13 g/cc. Calculate the values of ~~mean~~ ~~density~~

(1) mean density.

$$(P_m) = \frac{P_1 + P_2 + P_3 + P_4 + P_5}{5}.$$

$$= \frac{1.03 + 1.12 + 0.92 + 1.05 + 1.13}{5}$$

$$= \frac{5.39}{5} = 1.058 \approx 1.06 \text{ g/cc}$$

(11) absolute error -

$\Delta P_1 = P_m - P_1 = 1.06 - 1.03 = 0.03$ g/cc.

$\Delta P_2 = P_m - P_2 = 1.06 - 1.12 = -0.06$ g/cc

$\Delta P_3 = P_m - P_3 = 1.06 - 0.92 = 0.14$ g/cc

$\Delta P_4 = P_m - P_4 = 1.06 - 1.05 = 0.01$ g/cc

$\Delta P_5 = P_m - P_5 = 1.06 - 1.13 = -0.07$ g/cc

(111) mean absolute error =

$$\Delta \bar{P} = \frac{|\Delta P_1| + \cdots + |\Delta P_5|}{5}$$

$$= \frac{|0.03| + |-0.06| + |0.14| + |0.01| + |-0.07|}{5}$$

$$= \frac{0.31}{5} = 0.06 \text{ g/cc}.$$

(IV) $RE = \frac{\Delta \bar{P}}{P_m} = \frac{0.06}{1.06} = 0.056 \approx 0.06$ g/cc.

(5) The resistance of a wire as measured in an experiment was found to be 10·3 Ω, 10·7 Ω, 9·8 Ω 10·4 Ω and 9·6 Ω. Calculate.

(I) mean value of resistance →

$$R_m = \frac{R_1 + R_2 + R_3 + R_4 + R_5}{5}$$

$$= \frac{10.3 + 10.7 + 9.8 + 10.4 + 9.6}{5}$$

$$= \frac{50.8}{5} = 10.16$$

$$\approx 10.2\ \Omega.$$

(II) absolute error →

$\Delta R_1 = R_m - R_1 = 10.2 - 10.3 = -0.1\ \Omega$

$\Delta R_2 = R_m - R_2 = 10.2 - 10.7 = -0.5\ \Omega$

$\Delta R_3 = R_m - R_3 = 10.2 - 9.8 = 0.4\ \Omega$

$\Delta R_4 = R_m - R_4 = 10.2 - 10.4 = -0.2\ \Omega$

$\Delta R_5 = R_m - R_5 = 10.2 - 9.6 = 0.6\ \Omega$

(III) mean absolute error →

$$\overline{\Delta R_m} = \frac{|\Delta R_1| + \cdots + |\Delta R_5|}{5}$$

$$= \frac{|-0.1| + |-0.5| + |0.4| + |-0.2| + |0.6|}{5}$$

$$= \frac{1.8}{5} = 0.36$$

$$\approx 0.4\ \Omega.$$

(IV) fractional error $= \frac{\overline{\Delta R}}{R_m} = \frac{0.4}{10.2} = 0.03\ \Omega.$

(V) percentage error $= \frac{\overline{\Delta R}}{R_m} \times 100\% = 0.03 \times 100\%$

$= 3\%.$

⑥ The diameter of a wire as measured by a screw gauge was found to be 1·328 mm, 1·330 mm, 1·325 mm, 1·334 mm and 1·336 m.m. calculate,

(i) mean value of diameter.

$$L_m = \frac{L_1 + L_2 + L_3 + L_4 + L_5}{5} = \frac{1.328 + 1.330 + 1.325 + 1.334 + 1.336}{5}$$

$$= 1.3306 = \frac{6.653}{5}$$

$$= 1.3306$$

$$\simeq 1.331 \text{ mm}$$

(ii) absolute error →

$\Delta L_1 = L_m - L_1 = 1.331 - 1.328 = 0.003$ m.m

$\Delta L_2 = L_m - L_2 = 1.331 - 1.330 = 0.001$ m.m.

$\Delta L_3 = L_m - L_3 = 1.331 - 1.325 = 0.006$ m.m.

$\Delta L_4 = L_m - L_4 = 1.331 - 1.334 = -0.003$ m.m.

$\Delta L_5 = L_m - L_5 = 1.331 - 1.336 = -0.005$ m.m.

(iii) fractional mean absolute error =

$$\Delta \bar{L} = \frac{|\Delta L_1| + \cdots + |\Delta L_5|}{5}$$

$$= \frac{|0.003| + |0.001| + |0.006| + |-0.003| + |-0.005|}{5}$$

$$= \frac{0.018}{5} = 0.0036$$

$$\simeq 0.004 \text{ m.m.}$$

(iv) fractional error $= \frac{\Delta \bar{L}}{L_m} = \frac{0.004}{1.331} = 0.003$ m.m.

(v) $PE = \frac{\Delta \bar{L}}{L_m} \times 100\% = 0.003 \times 100\%$

$= 0.3\%$

(7) Using a screw ~~gauge~~ gauge, the diameter of a metal rod was measured. The observations are given as follows: 0.39 mm, 0.38 mm, 0.37 mm, 0.41 mm, 0.38 mm, 0.38 mm, 0.37 mm, ~~0.40~~ 0.40 mm, 0.39 mm. calculate -

(i) the most accurate value of the diameter -

$$(P_m) = \frac{(P_1) + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9}{9}.$$

$$= \frac{0.39 + 0.38 + 0.37 + 0.41 + 0.38 + 0.38 + 0.37 + 0.40 + 0.39}{9}$$

$$= \frac{3.47}{9} = 0.385 \approx 0.39 \text{ m.m}$$

(ii) ∴ absolute error ⇒

$\Delta P_1 = P_m - P_1 = 0.39 - 0.39 = 0$ ✓

$\Delta P_2 = P_m - P_2 = 0.39 - 0.38 = 0.01$ ✓

$\Delta P_3 = P_m - P_3 = 0.39 - 0.37 = 0.02$ ✓

$\Delta P_4 = P_m - P_4 = 0.39 - 0.41 = -0.02$ ✓

$\Delta P_5 = P_m - P_5 = 0.39 - 0.38 = 0.01$

$\Delta P_6 = P_m - P_6 = 0.39 - 0.38 = 0.01$

$\Delta P_7 = P_m - P_7 = 0.39 - 0.37 = 0.02$

$\Delta P_8 = P_m - P_8 = 0.39 - 0.40 = -0.01$ ✓

$\Delta P_9 = P_m - P_9 = 0.39 - 0.39 = 0$

∴ mean absolute error ⇒

$\Delta \bar{P} = |P_1| + \cdots + |\Delta P_9|$

$= \frac{0.39 + |0| + |0.01| + |0.02| + |-0.02| + |0.01| + |0.01| + |0.02| + |-0.01| + |0|}{9}$

$= \frac{0.09}{9} = 0.01$ m. $\frac{0.1}{9} = 0.011$ m.

$= 0.011$ m.

(ii)

(iii)

(8)

val

1.2

(1)

Sol

(ii)

Δ

Δ

Δ

Δ

Δ

Δ

Δ

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(ii) $RE = \frac{\Delta \bar{P}}{P_m} = \frac{0.004}{0.39} =$ ~~0.0103~~

$= \frac{0.004}{0.39} = 0.010$ ~~0.010~~ $= \frac{0.011}{0.39} = 0.028$

iii) $PE = \frac{\Delta \bar{P}}{P_m} \times 100\%$

$= \frac{0.028}{0.028} \times 100\%$

$= 1\%$ ~~1%~~ 2.8%

— x — x — — x —

(8) A student performs an experiment and found following values of the refractive index of 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30, 1.33. Find (i) mean value of refractive index.

Sol: $R_m = \frac{R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8}{8}$

$= \frac{1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30 + 1.33}{8}$

$= \frac{10.62}{8} = 1.327 \approx 1.33.$

(ii) The absolute error $\Rightarrow$

$\Delta R_1 = R_m - R_1 = 1.33 - 1.29 = 0.04$

$\Delta R_2 = R_m - R_2 = 1.33 - 1.33 = 0$

$\Delta R_3 = R_m - R_3 = 1.33 - 1.34 = -0.01$

$\Delta R_4 = R_m - R_4 = 1.33 - 1.35 = -0.02$

$\Delta R_5 = R_m - R_5 = 1.33 - 1.32 = 0.01.$

$\Delta R_6 = R_m - R_6 = 1.33 - 1.36 = -0.03$

$\Delta R_7 = R_m - R_7 = 1.33 - 1.30 = 0.03.$

$\Delta R_8 = R_m - R_8 = 1.33 - 1.33 = 0$

(ii) the mean absolute error =

$$\Delta \bar{R} = \frac{|\Delta R_1| + \ldots\ldots\ldots + |\Delta R_8|}{8}$$

$$= \frac{|0.04| + |0| + |-0.01| + |-0.02| + |0.01| + |-0.03| + |0.03| + |0|}{8}$$

$$= \frac{0.14}{8} = 0.017.$$

$$\approx 0.02.$$

(iii) $RE = \frac{\Delta \bar{R}}{R_m} = \frac{0.02}{1.33} = 0.150.$

$= 0.15.$

(iv) $PE = \frac{\Delta \bar{R}}{R_m} \times 100\%$

$= 0.15 \times 100\%$

$= 15\%$.