Question - Answer solution

1. The diameter of a wire measured in an experiment was 0.022 cm, 0.023 cm, 0.026 cm 0.025 cm, 0.024 cm and 0.025 cm.

Find

i) the mean value of diameter

ii) absolute error in a each measurement

iii) Percentage error.

Sol Let,

$D_1 = 0.022$

$D_2 = 0.023$

$D_3 = 0.026$

$D_4 = 0.025$

$D_5 = 0.024$

$D_6 = 0.025$

Now,

$$Dm = \frac{D_1 + \dots + D_6}{6} = \frac{0.145}{6} \approx 0.124.$$

So,

$\Delta D_1 = Dm - D_1 = 0.124 - 0.022 = +0.102$

$\Delta D_2 = Dm - D_2 = 0.124 - 0.023 = +0.101$

$\Delta D_3 = Dm - D_3 = 0.124 - 0.026 = +0.098$

$\Delta D_4 = Dm - D_4 = 0.124 - 0.025 = +0.099$

$\Delta D_5 = Dm - D_5 = 0.124 - 0.024 = +0.1$

$\Delta D_6 = Dm - D_6 = 0.124 - 0.025 = +0.099.$

Now,

$$R/M = \Delta R = \frac{|\Delta R_1| + \dots + |\Delta R_6|}{6}$$

$$= \frac{0.621}{6} = 0.10 \text{ m}$$

$$PE = \frac{0.10}{0.12} = [illegible] \times 100\%$$

2. Refractive index of a flint glass (μ) was measured in an experiment and was found to be 1.655, 1.667, 1.666, 1.659 1.669 and 1.654. Find

i) the mean value of μ
ii) mean absolute error
iii) Relative error
iv) Percentage error.

Sol: Let,

$\mu_1 = 1.655$
$\mu_2 = 1.667$
$\mu_3 = 1.666$
$\mu_4 = 1.659$
$\mu_5 = 1.669$
$\mu_6 = 1.654$.

Now,

$$\mu_m = \frac{\mu_1 + \ldots\ldots \mu_6}{6} = \frac{9.97}{6} = 1.66$$

So,

$\Delta\mu_1 = \mu_m - \mu_1 = 1.66 - 1.665 = -0.005$
$\Delta\mu_2 = \mu_m - \mu_2 = 1.66 - 1.667 = -0.007$
$\Delta\mu_3 = \mu_m - \mu_3 = 1.66 - 1.666 = -0.006$
$\Delta\mu_4 = \mu_m - \mu_4 = 1.66 - 1.659 = 0.001$
$\Delta\mu_5 = \mu_m - \mu_5 = 1.66 - 1.669 = -0.009$
$\Delta\mu_6 = \mu_m - \mu_6 = 1.66 - 1.654 = 0.006$

Now,

$$\Delta V = \frac{|\Delta V_1| + \ldots\ldots + |\Delta V_6|}{6}$$

$$= \frac{-0.02}{6} = \frac{-0.02 \times 100}{6100} = \frac{2}{600}$$

$$\approx 0.4$$

Now,

Result $= a = (a_m \pm \Delta a)$

$= (V_m \pm \Delta V)$

$= (1.66 \pm 0.4)$

So, Relative error $= \frac{\Delta V}{V_m} = \frac{0.4}{1.66} = 0.24.$

Percentage error $= \frac{\Delta V}{V_m} = \frac{0.4}{1.66} \times 100$

$= 24\%.$

3. While determining the time period of oscillation of a simple pendulum, the readings from various measurements are 1.73s, 1.62s, 1.52s, 1.45s and 1.83s. Calculate the value of mean of time period, absolute error, mean absolute error and relative error

Let,

$S_1 = 1.73s$

$S_2 = 1.62s$

$S_3 = 1.52s$

$S_4 = 1.45s$

$S_5 = 1.83s$

Now,

$$\frac{S_1 + \ldots\ldots S_5}{5} = \frac{8.15}{5} = 6.68$$

so,

$\Delta D_1 = 5.35 - 1.03 = 4.32$

$\Delta D_2 = 5.35 - 1.22 = 4.13$

$\Delta D_3 = 5.35 - 0.92 = 4.43$

$\Delta D_4 = 5.35 - 1.05 = 4.3$

$\Delta D_5 = 5.35 - 1.13 = 4.22$

wow,

$$\overline{\Delta D} = \frac{|\Delta D_1| + \ldots\ldots |\Delta D_5|}{5}$$

$$= 21.4$$

Hence,

Result = $(am \pm \overline{\Delta D})$

$= (5.35 \pm 21.4)$

Therefore,

relative error = $\frac{\overline{\Delta D}}{Dm} = \frac{21.4}{5.35} = 4$

5) The resistance of a wire as measured in an experiment was found to be 10.3Ω 10.7Ω, 9.8Ω, 10.4Ω & 9.6Ω. Calculate

i) mean value of resistance

ii) absolute error in each measurement

iii) mean absolute error

iv) fractional error

v) Percentage error.

Sol Let,

$R_1 = 10.3\Omega$

$R_2 = 10.1\Omega$

$R_3 = 9.8\Omega$

$R_4 = 10.4\Omega$

$R_5 = 9.6\Omega$.

Now,

$$\frac{R_1 + \ldots\ldots R_5}{5} = \frac{50.8}{5} = 10.16$$

So,

$\Delta R_1 = 10.16 - 10.3 = -0.14\ \Omega$
$\Delta R_2 = 10.16 - 10.7 = -0.54\ \Omega$
$\Delta R_3 = 10.16 - 9.8 = 0.36\ \Omega$
$\Delta R_4 = 10.16 - 10.4 = -0.24\ \Omega$
$\Delta R_5 = 10.16 - 9.6 = 0.56\ \Omega$

Now,

$$\overline{\Delta R} = \frac{|\Delta R_1| + \ldots\ldots |\Delta R_5|}{5}$$

$$= \frac{0}{5} \approx 0.$$

Now,

Result $= (am \pm \overline{\Delta R})$
$= (10.16 \pm 0)$

So,

Fractional error $= \frac{\overline{\Delta R}}{Rm} = \frac{0}{10.16} = 0.$

Finally,

Percentage error $= \frac{0}{10.16} \times 100 = 0\%.$

6. The diameter of a wire a measured by a screw gauge was found to be 1.328 mm 1.330 mm, 1.325 mm, 1.334 mm & 1.336 mm. calculate

i) mean value of diameter
ii) absolute value of in each error
iii) mean absolute error
iv) fractional error
v) percentage error.

Let,

$D_1 = 1.328$ mm
$D_2 = 1.330$ mm
$D_3 = 1.325$ mm
$D_4 = 1.334$ mm
$D_5 = 1.336$ mm.

Now,

$$D_m = \frac{D_1 + \ldots\ldots D_5}{5} = 6.653$$

$$\approx \frac{6.653}{5} = 1.33$$

So,

$\Delta D_1 = 1.33 - 1.328 = 0.002$
$\Delta D_2 = 1.33 - 1.330 = 0$
$\Delta D_3 = 1.33 - 1.325 = 0.005$
$\Delta D_4 = 1.33 - 1.334 = -0.004$
$\Delta D_5 = 1.33 - 1.336 = -0.006$

Now,

$$\overline{\Delta D} = \frac{|\Delta D_1| + \ldots\ldots |\Delta D_5|}{5}$$

$$= -0.003$$

$$= -0.0006$$

so, Fractional error = $\frac{\Delta D}{Dm}$

$= \frac{-0.0006}{1.33}$

so,

Percentage error

$= \frac{-0.0006}{1.33} \times 100$

$= 0.045\%$

7) Using a screw gauge, the diameter of a metre rod was measured. The observations are given as follows: 0.39 mm, 0.38 mm, 0.37 mm, 0.41 mm, 0.38 mm, 0.38 mm, 0.37 mm, 0.40 mm, 0.39 mm

Calculate,

i) the most accurate value of diameter
ii) the relative error
iii) the percentage error.

Given,

$D_1 = 0.39$ mm
$D_2 = 0.38$ mm
$D_3 = 0.37$ mm
$D_4 = 0.41$ mm
$D_5 = 0.38$ mm
$D_6 = 0.38$ mm
$D_7 = 0.37$ mm
$D_8 = 0.40$ mm
$D_9 = 0.39$ mm

Now,

$$D_m = \frac{D_1 + D_2 \ldots\ldots D_9}{9}$$

$$= 3.47$$

$$= 0.38$$

So,

$\Delta D_1 = 0.38 - 0.39 = -0.01$

$\Delta D_2 = 0.38 - 0.38 = 0$

$\Delta D_3 = 0.38 - 0.37 = 0.01$

$\Delta D_4 = 0.38 - 0.41 = -0.03$

$\Delta D_5 = 0.38 - 0.38 = 0$

$\Delta D_6 = 0.38 - 0.38 = 0$

$\Delta D_7 = 0.38 - 0.37 = 0.01$

$\Delta D_8 = 0.38 - 0.40 = -0.02$

$\Delta D_9 = 0.38 - 0.39 = -0.01$

So,

$$\overline{\Delta D} = \frac{|\Delta D_1| + \ldots\ldots |\Delta D_9|}{9}$$

$$= \frac{-0.05}{9} \simeq -0.005$$

So now,

$$\text{Relative error} = \frac{\overline{\Delta D}}{D_m}$$

$$= \frac{-0.005}{0.38} = -0.01$$

$$\text{Percentage error} = \frac{-0.005}{0.38} \times 100 = 1.31\%$$

8) A student performs an experiment and found following value of the refractive index of 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30, 1.33

Find the mean value of refractive index the mean absolute error. The refractive error and percentage error

Sol given,

$R_1 = 1.29$

$R_2 = 1.33$

$R_3 = 1.34$

$R_4 = 1.35$

$R_5 = 1.32$

$R_6 = 1.36$

$R_7 = 1.30$

$R_8 = 1.33$

So,

$$R_m = \frac{R_1 + \ldots\ldots R_8}{8} = 10.62$$

Now,

$\Delta R_1 = 10.62 - 1.29 = 9.33$

$\Delta R_2 = 10.62 - 1.33 = 9.29$

$\Delta R_3 = 10.62 - 1.34 = 9.28$

$\Delta R_4 = 10.62 - 1.35 = 9.27$

$\Delta R_5 = 10.62 - 1.32 = 9.3$

$\Delta R_6 = 10.62 - 1.36 = 9.26$

$\Delta R_7 = 10.62 - 1.30 = 9.32$

$\Delta R_8 = 10.62 - 1.33 = 9.29$

Where,

$$\overline{\Delta R} = \frac{|\Delta R_1| + \ldots\ldots |\Delta R_8|}{8}$$

$$= \frac{7/4}{3/4} \quad \frac{74.34}{8}$$

$$\approx 9.29$$

Now,

$$\frac{\Delta R1}{R1m} = \frac{9.29}{10.62}$$

$$= 0.87$$

Hence,

Percentage error $= \frac{9.29}{10.62} \times 100$

$= 87\%$.