

Error Calculation

Q: 1) ~~The diameter of a wire~~

Ans Let,

$$D_1 = 0.022 \text{ cm}$$

$$D_2 = 0.023 \text{ cm}$$

$$D_3 = 0.026 \text{ cm}$$

$$D_4 = 0.025 \text{ cm}$$

$$D_5 = 0.024 \text{ cm}$$

$$D_6 = 0.025 \text{ cm}$$

$$\therefore D_m = \frac{D_1 + D_2 + \dots + D_6}{6}$$

$$\neq \frac{0.022 + 0.023 + 0.026 + 0.025 +$$

$$= \frac{0.024 + 0.025}{6}$$

$$= \frac{0.145}{6} = 0.02416$$

$$\approx 0.024 \text{ cm}$$

$$\Delta D_1 = D_m - D_1 = 0.024 - 0.022 = 0.002 \text{ cm}$$

$$\Delta D_2 = D_m - D_2 = 0.024 - 0.023 = 0.001 \text{ cm}$$

$$\Delta D_3 = D_m - D_3 = 0.024 - 0.026 = -0.002 \text{ cm}$$

$$\Delta D_4 = D_m - D_4 = 0.024 - 0.025 = -0.001 \text{ cm}$$

$$\Delta D_5 = D_m - D_5 = 0.024 - 0.024 = 0 \text{ cm}$$

$$\Delta D_6 = D_m - D_6 = 0.024 - 0.025 = -0.001 \text{ cm}$$

$$\therefore \Delta \bar{D} = \frac{|\Delta D_1| + |\Delta D_2| + \dots + |\Delta D_6|}{6}$$

$$= \frac{0.007}{6}$$

$$= 0.001 \text{ cm}$$

$$\therefore \text{percentage error} = \frac{\Delta \bar{D}}{D_m} \times 100\%$$

$$= \frac{0.001}{0.024} \times 100\%$$

$$= \frac{1}{24} \times 100\%$$

$$= \frac{25}{6} \% = 4.16\%$$

$$= 4.2\%$$

a) 2) Ans

Let,

$$\mu_1 = 1.655$$

$$\mu_2 = 1.667$$

$$\mu_3 = 1.666$$

$$\mu_4 = 1.659$$

$$\mu_5 = 1.669$$

$$\mu_6 = 1.654$$

$$\therefore \mu_m = \frac{\mu_1 + \mu_2 + \dots + \mu_6}{6}$$

$$= \frac{\cancel{9.960}^7}{6} \quad \frac{9.970}{6}$$

$$\approx 1.661$$

$$\Delta \mu_1 = \mu_m - \mu_1 = 1.661 - 1.655 = 0.006$$

$$\Delta \mu_2 = \mu_m - \mu_2 = 1.661 - 1.667 = -0.006$$

$$\Delta \mu_3 = \mu_m - \mu_3 = 1.661 - 1.666 = -0.005$$

$$\Delta \mu_4 = \mu_m - \mu_4 = 1.661 - 1.659 = 0.002$$

$$\Delta \mu_5 = \mu_m - \mu_5 = 1.661 - 1.669 = -0.008$$

$$\Delta \mu_6 = \mu_m - \mu_6 = 1.661 - 1.654 = 0.007$$

$$\therefore \text{Mean absolute error, } \Delta \bar{\mu} = |\Delta \mu_1| + |\Delta \mu_2| + \dots$$

$$\frac{|\Delta \mu_6|}{6}$$

$$= \frac{0.034}{6}$$

$$= 0.00566$$

$$\approx 0.006$$

$$\therefore \text{Relative error, RE} = \frac{\Delta \bar{\mu}}{\mu_m}$$

$$= \frac{0.006}{1.661}$$

$$= \frac{6}{1661}$$

$$= 0.0036$$

$$\approx 0.004$$

$$\therefore \text{percentage error, \%E} = \frac{\Delta \bar{\mu}}{\mu_m} \times 100\%$$

$$= 0.004 \times 100\%$$

$$= 0.4\%$$

2) 3) Ans

Let,

$$T_1 = 1.73 \text{ s}$$

$$T_2 = 1.62 \text{ s}$$

$$T_3 = 1.52 \text{ s}$$

$$T_4 = 1.45 \text{ s}$$

$$T_5 = 1.83 \text{ s}$$

$$\begin{aligned}\therefore T_m &= \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5} \\ &= \frac{1.73 + 1.62 + 1.52 + 1.45 + 1.83}{5} \\ &= \frac{8.15}{5} \\ &= 1.63 \text{ s}\end{aligned}$$

$$\therefore \Delta T_1 = 1.63 - 1.73 = -0.10$$

$$\Delta T_2 = 1.63 - 1.62 = 0.01$$

$$\Delta T_3 = 1.63 - 1.52 = 0.11$$

$$\Delta T_4 = 1.63 - 1.45 = 0.18$$

$$\Delta T_5 = 1.63 - 1.83 = -0.20$$

∴ Mean absolute error,

$$\begin{aligned}\Delta \bar{T} &= \frac{|\Delta T_1| + |\Delta T_2| + \dots + |\Delta T_5|}{5} \\ &= \frac{0.60}{5} \\ &= 0.12 \text{ s.}\end{aligned}$$

∴ Relative error, RE =

$$\begin{aligned}\frac{\Delta \bar{T}}{T_m} &= \frac{0.12 \cancel{\text{s}}}{1.63 \cancel{\text{s}}} = \frac{12}{163} = 0.073 \\ &\approx 0.07\end{aligned}$$

Q) 4) Ans

Let,

$$P_1 = 1.03 \text{ g/cc}$$

$$P_2 = 1.12 \text{ g/cc}$$

$$P_3 = 0.92 \text{ g/cc}$$

$$P_4 = 1.05 \text{ g/cc}$$

$$P_5 = 1.13 \text{ g/cc}$$

$$\begin{aligned}
 \therefore \text{Mean value, } \rho_m &= \frac{\rho_1 + \rho_2 + \dots + \rho_5}{5} \\
 &= \frac{1.03 + 1.12 + 0.92 + 1.05 + 1.13}{5} \\
 &= \frac{5.25}{5} \\
 &= 1.05 \text{ g/cc}
 \end{aligned}$$

$$\therefore \Delta \rho_1 = 1.05 - 1.03 = 0.02 \text{ g/cc}$$

$$\Delta \rho_2 = 1.05 - 1.12 = -0.07 \text{ g/cc}$$

$$\Delta \rho_3 = 1.05 - 0.92 = 0.13 \text{ g/cc}$$

$$\Delta \rho_4 = 1.05 - 1.05 = 0 \text{ g/cc}$$

$$\Delta \rho_5 = 1.05 - 1.13 = \cancel{0.02} - 0.08 \text{ g/cc}$$

$$\begin{aligned}
 \therefore \text{Mean absolute error, } \Delta \bar{\rho} &= \frac{|\Delta \rho_1| + \dots + |\Delta \rho_5|}{5} \\
 &= \frac{0.02 + 0.07 + 0.13 + 0.08}{5} \\
 &= \frac{0.30}{5} \\
 &= 0.06 \text{ g/cc}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Relative error, RE} &= \frac{\Delta \bar{P}_t}{P_m} \\
 &= \frac{0.06}{1.05} \\
 &= \frac{6}{105} = 0.057 \\
 &= \underline{\underline{0.06}}
 \end{aligned}$$

a) 5) Ans

Let,

$$R_1 = 10.3 \Omega$$

$$R_2 = 10.7 \Omega$$

$$R_3 = 9.8 \Omega$$

$$R_4 = 10.4 \Omega$$

$$R_5 = 9.6 \Omega$$

$$\text{Mean value, } R_m = \frac{R_1 + R_2 + \dots + R_5}{5}$$

$$= \frac{10.3 + 10.7 + 9.8 + 10.4 + 9.6}{5}$$

$$= \frac{50.8}{5}$$

$$= 10.16 \approx 10.2 \Omega$$

$$\therefore \Delta R_1 = 10.2 - 10.3 = -0.1 \Omega$$

$$\Delta R_2 = 10.2 - 10.7 = -0.5 \Omega$$

$$\Delta R_3 = 10.2 - 9.8 = 0.4 \Omega$$

$$\Delta R_4 = 10.2 - 10.4 = -0.2 \Omega$$

$$\Delta R_5 = 10.2 - 9.6 = 0.6 \Omega$$

\therefore mean absolute error, $\Delta \bar{R}$

$$= \frac{|\Delta R_1| + \dots + |\Delta R_5|}{5}$$

$$= \frac{1.8}{5} = 0.36$$

$$\approx 0.4 \Omega$$

\therefore Fractional error

$$= \frac{\Delta \bar{R}}{R_m}$$

$$= \frac{0.4}{10.2} = \frac{4}{102} = \frac{2}{51} = 0.039$$

$$\approx 0.04$$

\therefore percentage error, PE

$$= \frac{\Delta \bar{R}}{R_m} \times 100\% = 0.04 \times 100\%$$

$$= 4\%$$

a) 6) Ans

Let,

$$D_1 = 1.328 \text{ mm}$$

$$D_2 = 1.330 \text{ mm}$$

$$D_3 = 1.325 \text{ mm}$$

$$D_4 = 1.334 \text{ mm}$$

$$D_5 = 1.336 \text{ mm}$$

$$\begin{aligned}\therefore \text{Mean value, } D_m &= \frac{D_1 + \dots + D_5}{5} \\ &= \frac{1.328 + 1.330 + 1.325 + 1.334 + 1.336}{5} \\ &= \frac{6.653}{5} = 1.3306 \\ &\approx 1.331 \text{ mm.}\end{aligned}$$

$$\therefore \Delta D_1 = 1.331 - 1.328 = 0.003 \text{ mm}$$

$$\Delta D_2 = 1.331 - 1.330 = 0.001 \text{ mm}$$

$$\Delta D_3 = 1.331 - 1.325 = 0.006 \text{ mm}$$

$$\Delta D_4 = 1.331 - 1.334 = -0.003 \text{ mm}$$

$$\Delta D_5 = 1.331 - 1.336 = -0.005 \text{ mm}$$

∴ Mean absolute error, $\Delta \bar{D}$

$$= \frac{|\Delta D_1| + |\Delta D_2| + \dots + |\Delta D_5|}{5}$$

$$= \frac{0.018}{5} = 0.0036$$

$$\approx 0.004 \text{ mm}$$

∴ Fractional error, RE

$$= \frac{\Delta \bar{D}}{D_m} = \frac{0.004}{1.331} = \frac{4}{1331} = 0.003$$

∴ percentage error, PE

$$= \frac{\Delta \bar{D}}{D_m} \times 100\% = 0.003 \times 100\% = 0.3\%$$

Q) 7) Ans

Let,

$$D_1 = 0.39 \text{ mm}$$

$$D_7 = 0.37 \text{ mm}$$

$$D_2 = 0.38 \text{ mm}$$

$$D_8 = 0.40 \text{ mm}$$

$$D_3 = 0.37 \text{ mm}$$

$$D_9 = 0.39 \text{ mm}$$

$$D_4 = 0.41 \text{ mm}$$

$$D_5 = 0.38 \text{ mm}$$

$$D_6 = 0.38 \text{ mm}$$

∴ Mean value, D_m

$$= \frac{D_1 + D_2 + \dots + D_9}{9}$$
$$= \frac{0.39 + 0.38 + 0.37 + 0.41 + 0.38 + 0.38 + 0.37 + 0.40 + 0.39}{9}$$

$$= \frac{3.47}{9} = 0.385$$
$$\approx 0.38 \text{ mm}$$

$$\therefore \Delta D_1 = 0.38 - 0.39 = -0.01$$

$$\therefore \Delta D_2 = 0.38 - 0.38 = 0$$

$$\Delta D_3 = 0.38 - 0.37 = 0.01$$

$$\Delta D_4 = 0.38 - 0.41 = -0.03$$

$$\Delta D_5 = 0.38 - 0.38 = 0$$

$$\Delta D_6 = 0.38 - 0.38 = 0$$

$$\Delta D_7 = 0.38 - 0.37 = 0.01$$

$$\Delta D_8 = 0.38 - 0.40 = -0.02$$

$$\Delta D_9 = 0.38 - 0.39 = -0.01$$

∴ Mean absolute error,

$$\begin{aligned}\Delta \bar{D} &= \frac{|\Delta D_1| + \dots + |\Delta D_9|}{9} \\ &= \frac{0.01 + 0.01 + 0.03 + 0.01 + 0.01 + 0.02}{9} \\ &= \frac{0.09}{9} \\ &= 0.01 \text{ mm}\end{aligned}$$

$$\begin{aligned}\therefore D &= (D_m \pm \Delta \bar{D}) \\ &= (0.38 \pm 0.01) \text{ mm}\end{aligned}$$

∴ Relative error, RE =

$$\frac{\Delta \bar{D}}{D_m} = \frac{0.01}{0.38} = \frac{1}{38} = 0.03$$

∴ Percentage error, PE

$$= \frac{\Delta \bar{D}}{D_m} \times 100\% = 0.03 \times 100\% = 3\%$$

Q) 8) Ans

Let,

$$M_1 = 1.29$$

$$M_2 = 1.33$$

$$M_3 = 1.34$$

$$M_4 = 1.35$$

$$M_5 = 1.32$$

$$M_6 = 1.36$$

$$M_7 = 1.30$$

$$M_8 = 1.33$$

∴ Mean value, M_m

$$= \frac{M_1 + \dots + M_8}{8}$$

$$= \frac{1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30 + 1.33}{8}$$

$$= \frac{10.62}{8} = 1.327$$

$$\approx 1.33$$

$$\therefore \Delta \mu_1 = 1.33 - 1.29 = 0.04$$

$$\Delta \mu_2 = 1.33 - 1.33 = 0$$

$$\Delta \mu_3 = 1.33 - 1.34 = -0.01$$

$$\Delta \mu_4 = 1.33 - 1.35 = -0.02$$

$$\Delta \mu_5 = 1.33 - 1.32 = 0.01$$

$$\Delta \mu_6 = 1.33 - 1.36 = -0.03$$

$$\Delta \mu_7 = 1.33 - 1.30 = 0.03$$

$$\Delta \mu_8 = 1.33 - 1.33 = 0$$

\therefore Mean absolute error, $\Delta \bar{\mu}$

$$= \frac{|\Delta \mu_1| + |\Delta \mu_2| + \dots + |\Delta \mu_8|}{8}$$

$$= \frac{0.04 + 0.01 + 0.02 + 0.01 + 0.03 + 0.03}{8}$$

$$= \frac{0.14}{8} = 0.0175 \approx 0.02$$

\therefore Relative error, RE

∴ percentage error, PE

$$= \frac{\Delta \bar{\mu}}{\mu_m} \times 100\% = 0.02 \times 100\% \\ = 2\%$$