Electric Potential

1. Prove that, $N/C = V/m$

Solⁿ:-

R.H.S $\frac{V}{m}$

We know that

$V = \frac{J}{C}$

Now, $\frac{J/C}{m}$

$= \frac{J}{C \times m}$

$= \frac{N \times m}{C \times m}$

$= \frac{N}{C} = L.H.S$//

—— × ——

(2) Calculate the workdone in moving a test ~~cat~~ charge $+q_0$ along a semi circular arc. with a source charge $+Q$ at the centre of the arc.

Solⁿ:-

Test charge $+q_0$ is moving along a semi circular arc.

$\Delta V = V_f - V_i$

$= 0$ ($\because$ It is a equipotential surface)

$\therefore W = q_0 (V_f - V_i)$

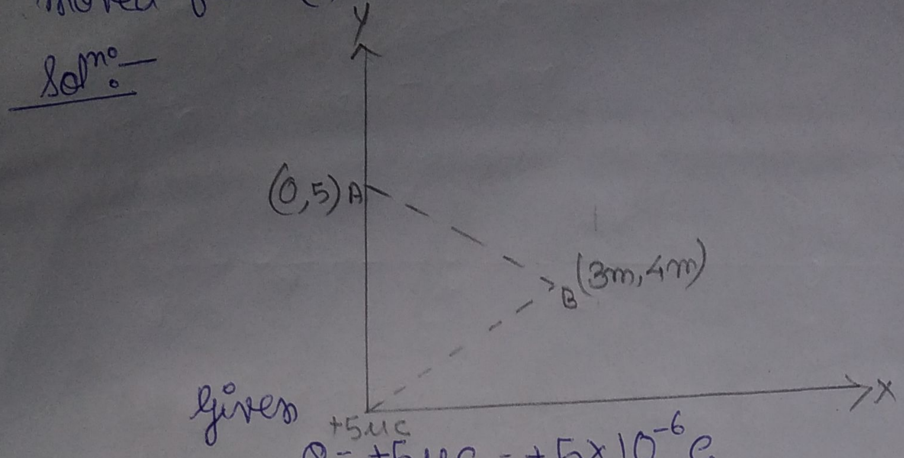
$= q_0 \times 0$

$= 0$

—— × ——

③ A point charge of +5μC is placed at the origin of co-ordinate system. An electron is to be moved from (0,5m) to (3m,4m). Find the work done.

Solⁿ:—



Given

$q = +5\mu C = +5\times10^{-6} C$

Potential at (0,5m)

$V_A = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r_A}$

$= 9\times10^{9} \times \frac{5\times10^{-6}}{5}$

$= 9\times10^{3}$ V

Potential at (3m,4m)

$V_B = \frac{1}{4\pi\epsilon_0} \frac{q}{r_B}$

$= 9\times10^{9} \times \frac{5\times10^{-6}}{\sqrt{3^2+4^2}}$

$= 9\times10^{9} \times \frac{5\times10^{-6}}{5}$

$= 9\times10^{3}$ V

∴ Charge of $e^- = -1.6\times10^{-19}$ C

$W = e^- [V_B - V_A]$

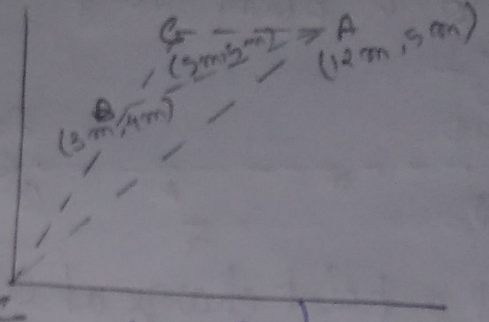
$= 1.6\times10^{-19} \times 0$

$= 0$

——— x ———

(4) A point charge of +8 μC is placed at the ~~point~~ origin of co-ordinate system. ~~A electron. Anon~~ Another charge of -2 nC is to be moved from (12m, 5m) to (3m, 4m) via (5m, 5m). Find the work done.

Solution:



Given

$Q = 8 \mu C$

$= 8 \times 10^{-6} C$

Let A = (12m, 5m); B = (3m, 4m); C = (5m, 5m)

Electric potential at A (12m, 5m)

$V_A = \frac{1}{4\pi \epsilon_0} \times \frac{q}{r_A}$

$= 9 \times 10^9 \times \frac{8 \times 10^{-6}}{\sqrt{12^2 + 5^2}}$

$= 9 \times 10^9 \times \frac{8 \times 10^{-6}}{13}$

$= \frac{72}{13} \times 10^3$

$= 5.53 \times 10^3 V$

Potential at B (3m, 4m)

$V_B = \frac{1}{4\pi \epsilon_0} \times \frac{q}{r_B}$

$= 9 \times 10^9 \times \frac{8 \times 10^{-6}}{\sqrt{3^2 + 4^2}}$

$= 9 \times 10^9 \times \frac{8 \times 10^{-6}}{5}$

$= 14.4 \times 10^3 V$

Charge of $e^- = -2 \times 10^{-9}$

$W_{A \to B \to C} = -2 \times 10^{-9} [14.4 \times 10^3 - 5.53 \times 10^3]$

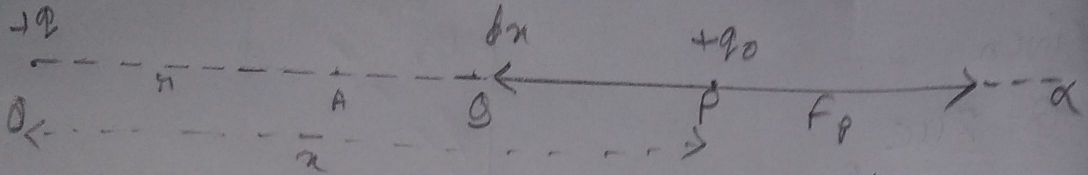
$= -2 \times 10^{-9} [14.4 - 5.53] 10^3$

$= -2 \times 8.87 \times 10^{-6}$

$= -17.74 \times 10^{-6}\ J$

Q5) Find an expression for the electric potential due to a point charge.

Solⁿ:—



Let a point charge at O due to which electric potential at A (OA = r) is

$V_A = \frac{W_{\alpha \to A}}{q_0}$ —— (i) [Where $W_{\alpha \to A}$ is the amount of work done in bringing $+q_0$ from α to A]

Let $OP = x$ $(r < x < \alpha)$

So repulsive force between $+q_0$ at P and $+q$ at O is

$$F_P = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{x^2} \quad \text{—— (ii)}$$

Let $\vec{PQ} = d\vec{x}$ (P & Q are very much closer to each other)

Amount of work done moving $+q_0$ from P to Q against repulsion is

$$dW_{P\to Q} = \vec{F_P}\cdot\vec{PQ}$$

$$= \vec{F}_P \cdot d\vec{x} = F_P\, dx \cos 180^\circ \quad [\because F_P \text{ \& } dx \text{ are in opposite dirⁿ}]$$

$$\Rightarrow dW_{P\to Q} = -\frac{1}{4\pi\epsilon_0}\frac{qq_0}{x^2} \quad \text{—— (iii)}$$

$$\therefore W_{\alpha\to A} = \int dW_{P\to Q} = \int_{\alpha}^{r} -\frac{1}{4\pi\epsilon_0}\frac{qq_0}{x^2}\,dx$$

$$= -\frac{1}{4\pi\epsilon_0}qq_0\int_{x}^{r}\frac{dx}{x^2}$$

$$= -\frac{1}{4\pi\epsilon_0}qq_0\left[-\frac{1}{x}\right]_{\alpha}^{r}$$

$$\Rightarrow \frac{W_{\alpha \to A}}{q_0} = \frac{1}{4\pi\epsilon_0} q q_0 \left[\frac{1}{r} - \frac{1}{\alpha}\right]$$

$$\Rightarrow V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{—— (iv)}$$