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Error Calculation - 01

Q:1) The percentage error in the distance (125.0 ± 0.5) cm is -

Ans:- (c) 0.4%

$\frac{0.5}{125} \times 100\% = 0.4\%$

Q.2) When a current of $(2.5 + 0.5)$ A flows through a wire, it develops a potential difference of $(20 + 1)$ V. The resistance of the wire is

Sol^n:- Here, $R = \frac{V}{I}$

$$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{1}{20} + \frac{0.5}{2.5} = \frac{1}{4}$$

$$\Rightarrow \Delta R = \frac{1}{4} \times R = \frac{1}{4} \times \frac{V}{I} = \frac{1}{4} \times \frac{20}{2.5}$$

$$\Rightarrow \Delta R = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{0.5} \Rightarrow \Delta R = \pm 2\ \Omega$$

and $R = \frac{V}{I} = \frac{20}{2.5} = \frac{20}{0.5} = 8\ \Omega$

$\therefore$ Resistance $= (8 \pm 2)\ \Omega$

Q.3) Two resistors of resistances $R_1 = (100 \pm 3)\ \Omega$ and $R_2 = (200 \pm 4)\ \Omega$ are connected in series. Find the equivalent resistance of the series connection.

Sol^{n}:- Given,

$R_1 = (100 \pm 3)\ \Omega$

$R_2 = (200 \pm 4)\ \Omega$

So, equivalent resistance $= (R_1 + R_2) \pm (\Delta R_1 + \Delta R_2)$

$= \{(100 + 200) \pm (3 + 4)\}\ \Omega$

$= (300 \pm 7)\ \Omega$

Q.4) The temperature at 7AM is $(27.0 \pm 0.5)^{\circ}C$ and at 11AM, it $(52.5 \pm 0.5)^{\circ}C$. Find the rise in temperature.

Sol^{n}:- Given Let

$T_1 = (27 \pm 0.5)^{\circ}C$ and $T_2 = (52.5 \pm 0.5)^{\circ}C$

So,

rise in temperature $= (T_2 - T_1) \pm (\Delta T_1 + \Delta T_2)$

$= (52.5 - 27) \pm (0.5 + 0.5)$

$= (25.5 \pm 1)^{\circ}C$

∴ Rise in temperature is $(25.5 \pm 1)^{\circ}C$.

Q.5) The mass of an object is (225 ± 0.05) g. Calculate % error in this measurement.

Sol^{n}:- Given,

$M = (225 \pm 0.05)$ g

Mass percentage error $= \frac{0.05}{225} \times 100\ \%$

$= 0.02\ \%$

Q.6) The length, breadth, and thickness of a block of metal were measured by a student in practical lab. The measurements are:

$l = (5.250 \pm 0.001)$ cm $\quad b = (3.450 \pm 0.001)$ cm,

$t = (1.740 \pm 0.001)$ cm

Find the volume of the block with percentage error in the measurement of the volume of the block.

Solⁿ:- Here,

$$V = l \times b \times h$$

$$\Rightarrow V = 5.250 \times 3.450 \times 1.740$$

$$= 31.51575 \approx 31.52 \text{ cm}^3$$

again,

$$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{0.001}{5.250} + \frac{0.001}{3.450} + \frac{0.001}{1.740}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{0.0002 + 0.0003 + 0.006}{1}$$

$$\Rightarrow \frac{\Delta V}{V} = 0.0011 \quad \Rightarrow \frac{\Delta V}{V} = 0.0011 \times V$$

$$\Rightarrow \Delta V = 0.0011 \times 31.52$$

$$\Rightarrow \Delta V = 0.034 \text{ cm}^3$$

∴ Volume with error = $(31.52 \pm 0.034)\ cm^3$

Thus, percentage error = 0.10%

Q.7) The refractive index (μ) of a glass slab were found to have values 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30 and 1.33. Find the mean absolute value of μ, the percentage error. Express the value of μ with error correctio

Sol^n:- The mean value of μ is

~~(1.33+1.29+~~ 1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30)/8

= 10.62/8 = 1.32

∴ $\mu_m = 1.32$

Now, absolute error.

$\Delta\mu_1 = 1.32 - 1.29 = 0.03$

$\Delta\mu_2 = 1.32 - 1.33 = -0.01$

$\Delta\mu_3 = 1.32 - 1.34 = -0.02$

$\Delta\mu_4 = 1.32 - 1.35 = -0.03$

$\Delta\mu_5 = 1.32 - 1.36 = -0.04$

$\Delta\mu_6 = 1.32 - 1.32 = 0$

$\Delta\mu_7 = 1.32 - 1.30 = 0.02$

$\Delta\mu_8 = 1.32 - 1.33 = -0.01$

and

$$\Delta\bar{\mu} = \left(|\Delta\mu_1| + |\Delta\mu_2| + \cdots |\Delta\mu_8|\right) / 8$$

so,

$\Delta\bar{4} = (0.03 + 0.01 + 0.02 + 0.03 + 0.04 + 0 + 0.02 + 0.01)/8$

$= 0.16/8 = 0.02$

∴ Mean absolute value of 4 is 0.02.

The percentage error of 4 is $\frac{0.02}{1.32} \times 100\%$

$= 1.5\%$

and the value of 4 with error correction is (1.32 ± 0.02)

Q.8) If the volume of a cylinder is $V = (0.785 \pm 0.008)\ cm^3$ and mass $m = (6.25 \pm 0.01)\ g$. Find the density and the percentage error in determination of density.

Soln:- We know,

$$\rho = \frac{m}{V}$$

$$\Rightarrow \rho = \frac{6.25\ g}{0.785\ cm^3} \Rightarrow \rho = 7.96\ g\,cm^{-3}$$

again, $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} \pm \frac{\Delta V}{V}$

$$\Rightarrow \frac{\Delta \rho}{7.96} = \frac{0.01}{6.25} + \frac{0.008}{0.785}$$

$$\Rightarrow \Delta\rho/7.96 = 0.002 + 0.010$$

$$\Rightarrow \Delta\rho/7.96 = 0.012$$

$$\Rightarrow \Delta\rho = 0.012 \times 7.96 = 0.09$$

∴ The density of the cylinder is $(7.96 \pm 0.09)\ g\ cm^{-3}$.

again,

Percentage error $= \frac{0.09}{7.96} \times 100$

$= 1.1\%$

Q.9) In Ohm's experiment, the values of an unknown resistance were found to be 4.12 Ω, 4.08 Ω, 4.22 Ω and 4.14 Ω. Calculate absolute error, relative error and percentage error in these measurement.

Sol^{n}: True value of these resistances are $= (4.12 + 4.08 + 4.22 + 4.14)/4$

$= 16.56/4 = 4.14\ \Omega$

Now, Absolute error of these resistances are

$R_1 = 4.14 - 4.12 = 0.02\ \Omega$

$R_2 = 4.14 - 4.08 = 0.06\ \Omega$

$R_3 = 4.14 - 4.22 = -0.08\ \Omega$

$R_4 = 4.14 - 4.14 = 0$

again, mean absolute error is

$\Delta\bar{R} = (0.02 + 0.06 + 0.08)/4$

$= 0.16/4 = 0.04\ \Omega$

also, relative error is

$$RE = \frac{\Delta\bar{R}}{R_m} = \frac{0.04}{4.14}\ \Omega$$

and percentage error

$$PE = \frac{0.04}{4.14} \times 100\%$$

$$= 0.96\ \%$$

Q.10) The acceleration of a car is given as, $a = (2 \pm 0.01)\ m/s^2$. Find the velocity of the car in a time period, $t = (10 \pm 0.1)s$. Also, find the percentage error in the measurement of velocity.

Sol^n:- We know,

$$a = \frac{v}{t} \Rightarrow v = a \times t$$

$$\Rightarrow v = (2 \times 10)\ m/s = 20\ m/s$$

$$\frac{\Delta a}{a} + \frac{\Delta t}{t} = \frac{\Delta v}{v}$$

$$\Rightarrow \frac{0.01}{2} + \frac{0.01}{10} = \frac{\Delta v}{20}$$

$$\Rightarrow \frac{0.05 + 0.01}{10} = \frac{\Delta v}{20}$$

$$\Rightarrow 0.06 \times 2 = \Delta v \quad \Rightarrow \Delta v = 0.12 \text{ m/s}$$

∴ Velocity of the car is (20 ± 0.12) m/s

So,

$$\text{percentage error} = \frac{0.12}{20} \times 100 = \frac{0.06}{0.12} \times 100$$

$$= 0.6\%$$