Electric Potential :: Paper - 01

1) Prove that, N/C = V/m.

Solⁿ :→

We know, $V = \frac{J}{C}$.

$$RHS = \frac{V}{m}$$

$$= \frac{J/C}{m}$$

$$= \frac{J}{C \times m}$$

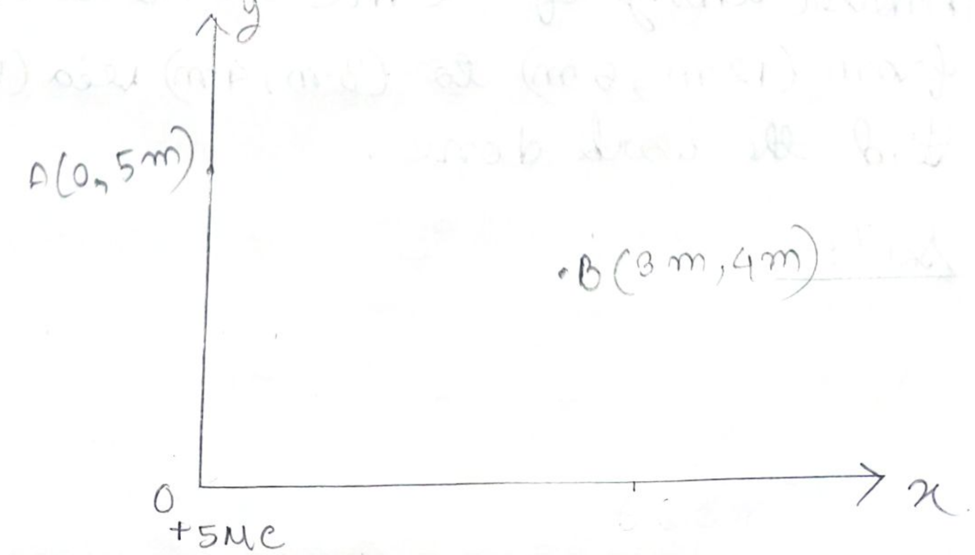
$$= \frac{N \times m}{C \times m} \quad [\because J = N \times m].$$

$$= \frac{N}{C}$$

$$= LHS.$$

3) A point charge of +5 μC is placed at the origin of co-ordinate system. An electron is to be moved from (0, 5 m), to (3 m, 4 m). Find work done.

Solⁿ :→



Given, $q = +5 \times 10^{-6} C$.

$q_0 = -1.6 \times 10^{-19} C$.

$r_A = 5 m$

$r_B = 5 m$

We know, $W_{A \to B} = q_0 (V_B - V_A)$.

Now,

$$V_A = 9 \times 10^9 \times \frac{q}{r_A} = 9 \times 10^9 \times \frac{5 \times 10^{-6}}{5}$$

$$= 9 \times 10^3 V$$

$$V_B = 9 \times 10^9 \times \frac{q}{r_B} = 9 \times 10^9 \times \frac{5 \times 10^{-6}}{5}$$

$$= 9 \times 10^3 V.$$

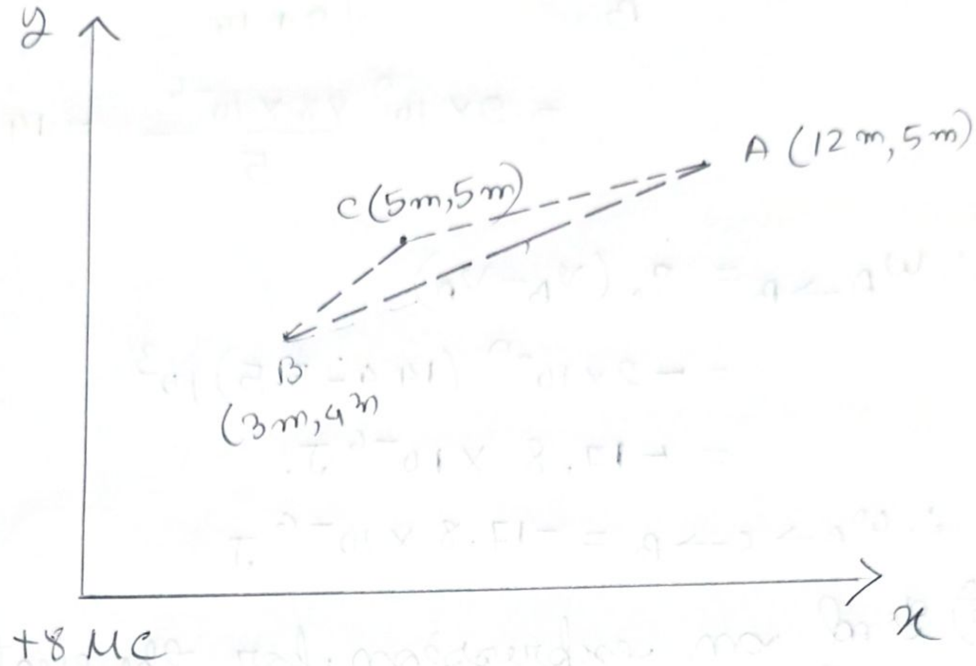
$$\therefore W_{A \to B} = q_0 (V_B - V_A)$$

$$= -1.6 \times 10^{-19} (9 - 9) \times 10^3$$

$$= 0$$

4) A point charge of +8 μC is placed at the origin of co-ordinate system. An electron Another charge of −2 mC is to be moved from (12 m, 5 m) to (3 m, 4 m) via (5 m, 5 m). Find the work done.

Solⁿ: →



Let, $A = (12m, 5m)$, $B = (3m, 4m)$, $C = (5m, 5m)$

given, $q = +8 \times 10^{-6}$ C

$q_0 = -2 \times 10^{9}$ C.

∵ workdone by conservative force depends on initial and final position and not on the path it travelled.

So, $W_{A \to C \to B} = W_{A \to B}$

$= q_0 [V_B - V_A]$.

$\therefore V_A = 9 \times 10^9 \frac{q}{r_A} = 9 \times 10^9 \times \frac{8 \times 10^{-6}}{\sqrt{144+25}}$.

$= \frac{9 \times 10^9 \times 8 \times 10^{-6}}{13}$.

$= 5.5 \times 10^3$ V.

$$V_B = 9 \times 10^9 \frac{q}{r_B} = 9 \times 10^9 \frac{\times 8 \times 10^{-6}}{\sqrt{9+16}}$$

$$= 9 \times 10^9 \frac{\times 8 \times 10^{-6}}{5} = 14.4 \times 10^3 \text{ V.}$$

$$\therefore W_{A \to B} = q_0 (V_B - V_A)$$

$$= -2 \times 10^{-9} (14.4 - 5.5) 10^3$$

$$= -17.8 \times 10^{-6} \text{ J.}$$

$$\therefore W_{A \to C \to B} = -17.8 \times 10^{-6} \text{ J.}$$

5) Find an expression for the electric potential due to a point charge.

Solⁿ :→

Let, the point charge +q be at O, due to which we have to find out electric potential at A, OA = r.

By the defⁿ of electric potential,

Electric potential at A.

$$V_A = \frac{W_{\alpha \to A}}{q_0} \longrightarrow (i)$$

[where $W_{\alpha \to A}$ is the amount of workdone in moving $+q_0$ from infinity to A].

Let, OP $= x$ ($r < x$ ∞).

Let the test charge $+q_0$ at P.

Repulsive force between $+q$ at O and $+q_0$ at P.

$$E_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{x^2} \text{ (towards right)} \longrightarrow \text{(ii)}$$

Let $\vec{PQ} = d\vec{x}$

The small amount of workdone in bringing $+q_0$ from P to Q

$$dw_{P\to Q} = \vec{F_p} \cdot \vec{PQ}$$

$\Rightarrow dw_{P\to Q} = F_p\, dx \cos 180°$ [$\vec{F_p}$ and $\vec{PQ}$ are in opposite direction].

$$\Rightarrow dw_{P\to Q} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{x^2}\, dx \longrightarrow \text{(iii)}.$$

$$\therefore w_{\infty\to A} = \int dw_{P\to Q} = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2}\, dx.$$

$$\Rightarrow w_{\infty\to A} = \frac{-1}{4\pi\epsilon_0} qq_0 \int_{\infty}^{r} \frac{dx}{x^2}.$$

$$\Rightarrow w_{\infty\to A} = -\frac{1}{4\pi\epsilon_0} q \left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow V_A = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\Rightarrow V_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \longrightarrow \text{(iv)}$$