Test - 1

Ans 1(a) (i) Pressure = Energy per unit volume

$[ML^{-1}T^{-2}]$ $[ML^{-1}T^{-2}]$

(b) dimensional formula of angle is $[M^0L^0T^0]$

SI unit = radian

(c) $F = Kv^2$

Taking dimensions of each term, we get

$[MLT^{-2}] = [K][LT^{-1}]^2$

$\Rightarrow [K] = [MLT^{-2}][L^2T^{-2}]$

$[K] = [ML^{-1}]$

(d) Mass (m) = 5.74 g

Volume (v) = 1.2 cm^3

Density = m/v

= 5.74/1.2 g/cm^3

= 4.78333 g/cm^3

$= 4783333.33 \text{ Kg/m}^3$

(c) Two limitation of dimensional analysis are —

(1) This method does not tell us how the dimensionless proportionality constant value.

(2) A formula containing trigonometric function, exponential function, and logarithmic function can not derive from it.

Ans. 2 (a) $v^2 = u^2 + 2as$, writing dimensional formula for each of them

$$[LT^{-1}]^2 = [LT^{-1}]^2 + [LT^{-2}][L]$$

$$[L^2T^{-2}] = [L^2T^{-2}] + [L^2T^{-2}]$$

∴ All the terms have same dimensional formula hence the equation is correct.

(b) We can add and structure only like quantities

dimension of P = dimension of $\frac{a}{v^2}$ — (i)

and dimension of v = dimensions of b from (i) — (ii)

Dimension of a = Dimension of P × Dimension of v^2 [a]

$= [M^1 L^{-1} T^{-2}] \times [L^3]^2$

$= [M^1 L^5 T^{-2}]$

unit of a = unit of P × Unit of v^2

$= \frac{N}{m^2} \times m^6 = N m^4$

From (ii), $[b] = [v] = [M^0 L^3 T^0]$

no unit of b = unit of $V = m^3$

(d) The given physical quantity $A = \frac{a^2 b^3}{c\sqrt{d}}$

Given: $\frac{\Delta a}{a} \times 100 = 1\%$, $\frac{\Delta b}{b} \times 100 = 3\%$

$\frac{\Delta c}{c} \times 100 = 2\%$ and $\frac{\Delta d}{d} \times 100 = 2\%$

Percentage error in A, $\frac{\Delta A}{A} \times 100 = 2\frac{\Delta a}{a} \times 100 + 3\frac{\Delta b}{b} \times 100 + \frac{\Delta c}{c} \times 100 + \frac{1}{2} \times \frac{\Delta d}{d} \times 100$

$\Rightarrow \frac{\Delta A}{A} \times 100 = 2 \times 1 + 3 \times 3 + 2 + \frac{1}{2} \times 2 = 14\%$

Percentage error in the qty quantity A is 14%.

Ans 3 (a) Let the relation be:

Frequency $N = K L^a f^b m^c$

where K is a dimensionless constant

The dimension of each quanty are:

~~$L = [M^0 L^0 T^2]$~~ $L = [M^0 L^1 T^0]$

$f = [M^1 L^1 T^{-2}]$

$m = [M^1 L^{-1} T^0]$

$v = [M^0 L^0 T^{-1}]$

Thus now equating the dimension on both side of the equation:

$$[M^0 L^0 T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^1 T^{-2}]^b [M^1 L^{-1} T^0]^c$$

thus, $b = \frac{1}{2}$

$$c = -\frac{1}{2}$$

$$a = -1$$

thus the formula of frequency is

$$\nu = \frac{k}{L}\sqrt{\frac{f}{m}}$$

(b) average length

$$\frac{2.48 + 2.46 + 2.49 + 2.50 + 2.52 + 2.43}{6}$$

$$n = 2.48\,m$$

~~mean absolute error~~

$$n_1 = 2.48 - 2.48 = 0$$

$$n_2 = 2.48 - 2.46 = 0.02$$

$n_3 = 2.48 - 2.49 = -0.01$

$n_4 = 2.48 - 2.50 = -0.02$

$n_5 = 2.48 - 2.52 = -0.04$

$n_6 = 2.48 - 2.43 = 0.05$

Mean absolute error

$$\frac{0 + 0.02 + (-0.01) + (-0.02) + (-0.04) + 0.05}{6}$$

$= 0$

~~Percentage error~~ Percentage error

$= 0 \times 100\%$

$= 0\%$

(d) If g is the period T

$T \propto p^a d^b E^c$

$T = k p^a d^b E^c$ .... ①

where k is a constant of proportionality and

dimensionless quantity:

inserting the dimensions of time, pressure, density and energy in equation (I) we get

$$[T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

equating powers of M, L, T on both sides we get,

$$0 = a + b + c \quad \cdots (II)$$

$$0 = -a - 3b + 2c \quad \cdots (III)$$

$$1 = -2a - 2c \quad \cdots (IV)$$

by solving (II) (III) (IV)

$$a = \frac{-5}{6} \quad b = \frac{1}{2} \quad \text{and} \quad c = \frac{1}{3}$$