NCERT-XII / Unit- 7 – Alternating Current

Alternating current The emf induced in the coil varies in magnitude and direction periodically, is called an alternating emf.

Mean value of AC

Mean value of an AC is defined as that value of steady current , which is constituted by that amount of charge flowing for the half cycle of the given AC. It is denoted by I_m

Let an alternating current $\mathbf{i = I_o \sin \omega t}$ flows through a circuit with T as time period .Its instantaneous value is , $i = dq / dt$

$$\Rightarrow dq = i\,dt = I_o \sin \omega t\,dt \ldots\ldots\ldots\ldots\ldots\ldots 1$$

Total charge flowing for the half cycle of the given AC

$$q = \int_0^q dq = \int_0^{\frac{T}{2}} I_o \sin \omega t\,dt$$

$$= -I_o\left[\frac{\cos \omega t}{\omega}\right]_0^{\frac{T}{2}} = -\frac{I_o}{\omega}\left[\cos \omega t\right]_0^{\frac{T}{2}} = -\frac{I_o}{\frac{2\pi}{T}}\left[\cos \frac{2\pi}{T}\cdot\frac{T}{2} - \cos \frac{2\pi}{T}\cdot 0\right] = -\frac{I_o T}{2\pi}\left[-1-1\right]$$

$$q = \frac{I_o T}{\pi} \quad \ldots\ldots\ldots (2)$$

So mean value of the given AC is

$\mathbf{I_m = q / (T/2)}$

$\mathbf{= 2 I_0 / \pi}$ **......................................3**

Thus, the mean value of an a.c is 0.638 times the peak value of the a.c.

In other words it is 63.8 % of the peak value.

NCERT-XII / Unit- 7 – Alternating Current

RMS value of a.c.

The rms value of alternating current is defined as that value of the steady current, which when passed through a resistor for a given time, will generate the same amount of heat as generated by an alternating current when passed through the same resistor for the same time.

It is denoted by I_{rms}.

When an alternating current **$i = I_o \sin \omega t$** flows through a resistor of resistance R, the amount of heat produced in the resistor in a small time dt is

$dH = i^2 R\, dt$ ……………1

The total amount of heat produced in the resistance in one complete cycle is

$$H = \int_0^T i^2 R\, dt = \int_0^T (I_o^2 \sin^2 \omega t) R\, dt$$

$$= I_o^2 R \int_0^T \left(\frac{1 - \cos 2\omega t)}{2} \right) dt$$

$$= \frac{I_o^2 R}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t . dt \right] = \frac{I_o^2 R}{2} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T = \frac{I_o^2 R}{2} \left[T - \frac{\sin 4\pi}{2\omega} \right]$$

$$H = \frac{I_o^2 R T}{2}$$

But this heat is also equal to the heat produced by rms value of AC in the same resistor (R) and in the same time (T),

$$\text{(i.e)} \ H = I_{rms}^2 RT$$

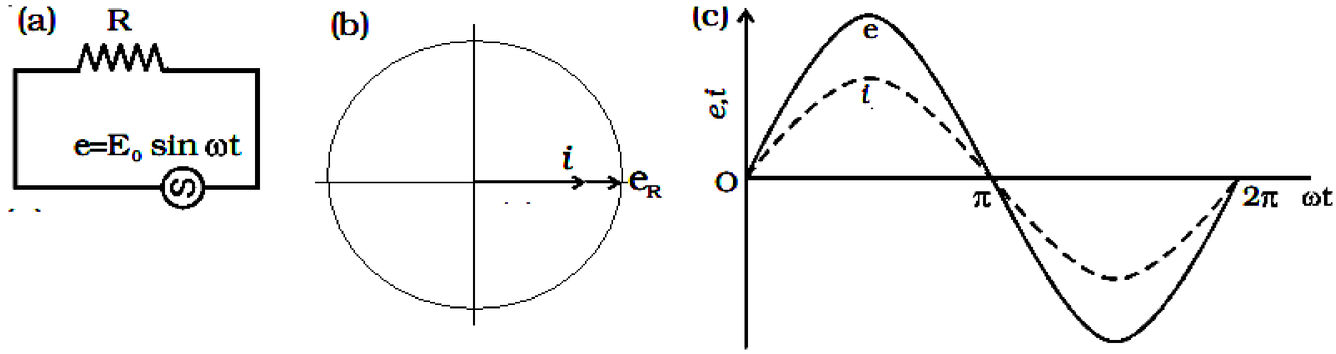
$$\therefore \ I_{rms}^2 RT = \frac{I_o^2 RT}{2}$$

$$I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707\, I_0$$

Thus, the rms value of an a.c is 0.707 times the peak value of the a.c. In other words it is 70.7 % of the peak value.

NCERT-XII / Unit- 7 – Alternating Current

AC Circuit with resistor



Let an alternating source of emf $e = E_0 \sin \omega t$ be connected across a resistor of resistance R.

In ideal condition , the potential drop across R must be equal to the applied emf.

so, $iR = E_0 \sin \omega t$

⇨ $i = (E_0 /R) \sin \omega t$

⇨ **$i = I_0 \sin \omega t$**……….(2)

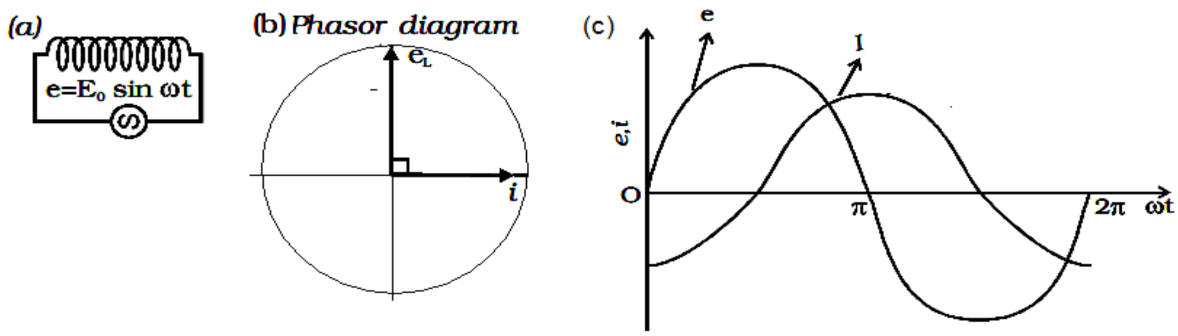
where $I_0 = E_0 /R$, is the peak value of a.c in the circuit.

So, it is seen that in a resistive circuit, the applied voltage and current are in phase with each other

Fig (b) is the phasor diagram showing the phase relationship between the current & the voltage.

NCERT-XII / Unit- 7 – Alternating Current

AC Circuit with an inductor



Let an alternating emf **$e = E_o \sin \omega t$** be applied to a pure inductor of inductance L. Due to self induction of the coil , a self induced emf is generated which opposes the applied voltage..

Induced emf $e' = -L .(di /dt)$

where L is the self inductance of the coil.

In an ideal inductor circuit induced emf is equal and opposite to the applied voltage.

Therefore $e = -e'$

$$E_o \sin \omega t = -\left(-L\frac{di}{dt}\right) = L\frac{di}{dt}$$

$$\Rightarrow di = \frac{E_o}{L} \sin \omega t \; dt$$

Integrating both the sides

$$\therefore i = \frac{E_o}{L}\int \sin \omega t \; dt = \frac{E_o}{L}\left[-\frac{\cos\omega t}{\omega}\right] = -\frac{E_o \cos\omega t}{\omega L}$$

$$i = \frac{E_o}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right)$$, $X_L = \omega L$ is knwon as inductive reactance

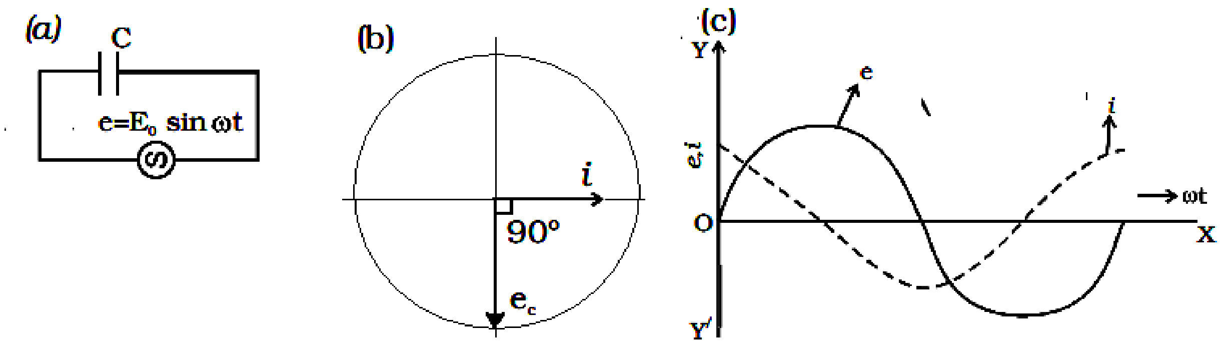
$$\boxed{i = I_o \cdot \sin\left(\omega t - \frac{\pi}{2}\right)}$$

So it is seen that in an a.c. circuit containing a pure inductor the current I lags behind the voltage e by the phase angle of π/2.

Or , the voltage across L leads the current by phase angle of π/2.

Fig b. gives the phasor diagram

AC Circuit with a capacitor



An alternating source of emf $\mathbf{e = E_0 \sin\omega t}$ is connected across a capacitor of capacitance C .

At any instant the potential difference across the capacitor will be equal to the applied emf

⇨ **e = q/C**, where q is the charge in the capacitor .

$$\text{But } i = \frac{dq}{dt} = \frac{d}{dt}(C_e)$$

$$i = \frac{d}{dt}(CE_0 \sin \omega t) = \omega CE_0 \cos \omega t = \frac{E_o}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = \frac{E_o}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right), \; X_C = \frac{1}{\omega C} \text{ is knwon as capacitive reactance}$$

$$\boxed{i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)}$$

So it is seen that **in an a.c. circuit with a capacitor, the current leads the voltage by a phase angle of π/2.**

In otherwords the emf lags behind the current by a phase angle of π/2.

Fig (b) represents phasor diagram

Inductive reactance

$X_L = \omega L = 2\pi\nu L$, where ν is the frequency of the a.c. supply

For d.c. $\nu = 0$; so $X_L = 0$

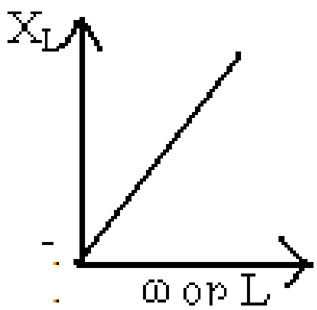
Thus a pure inductor offers zero resistance to d.c. But in an a.c. circuit the reactance of the coil increases with increase in frequency.

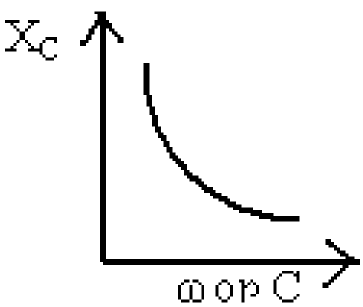
Capacitive reactance

$X_C = 1/\omega C = 1/ 2\pi\nu C$, where ν is the frequency of the a.c. supply

For d.c. $\nu = 0$; so $X_C = \propto$

Thus a pure capacitor offers infinite resistance to d.c. But in an a.c. the capacitive reactance varies inversely as the frequency of a.c. and also inversely as the capacitance of the capacitor.

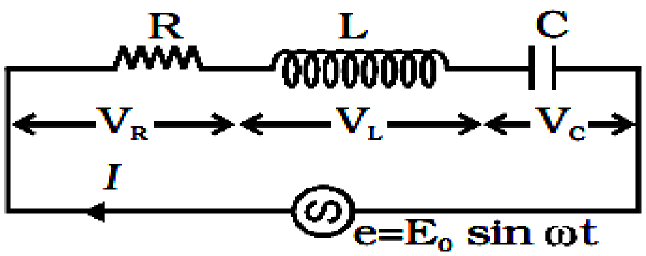


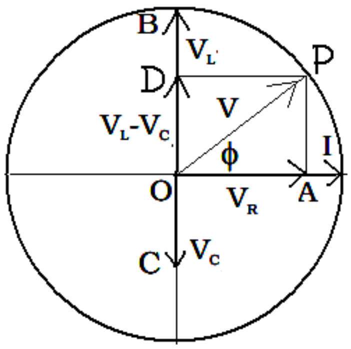


NCERT-XII / Unit- 7 – Alternating Current

LCR circuit

Let an alternating source of emf **e =E_0sinωt** be connected to a series combination of a resistor of resistance R, inductor of self inductance L and a capacitor of capacitance C as shown in figure below .





Let the current flowing through the circuit be I.

The voltage drop across different elements of the LCR circuit are

(i) across resistor is, $\mathbf{V_R = I\,R}$, V_R and I are in phase

(ii) across inductor is $\mathbf{V_L = I\,X_L}$, V_L is ahead of I by π/2

(iii) across capacitor is, $V_C = IX_C$, V_C lagging behind I by π/2

Representing the voltages across R , L and C by OA , OB and OC in the voltage phasor diagram as shown above , we have OD = $V_L - V_C$ As V_L and V_C are 180° out of phase with each other .

Completing rectangle OAPD , whose diagonal OP will give us resultant voltage in the LCR circuit .

$OP^2 = OA^2 + OD^2$

$V^2 = V_R^2 + (V_L - V_C)^2$.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 - (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2}$$

NCERT-XII / Unit- 7 – Alternating Current

The expression $\sqrt{R^2+(X_L-X_C)^2}$ is the net effective opposition offered by the combination of resistor, inductor and capacitor known as the impedance of the circuit and is represented by Z. Its unit is ohm.

The phase angle φ between the voltage and current is given by

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\text{net reactance}}{\text{resistance}}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

so , **$I = I_o \sin(\omega t + \varphi)$** is the instantaneous current flowing in the circuit.

Resonance

Resonance is the phenomenon by virtue of which , current in an LCR circuit is maximum .

It happens when the natural frequency of the LCR circuit is equal to the frequency of the energy source .

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

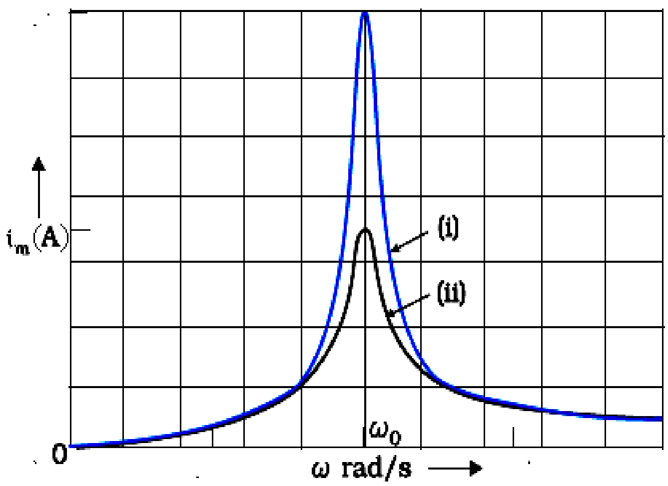
at a particular frequency ω_0,

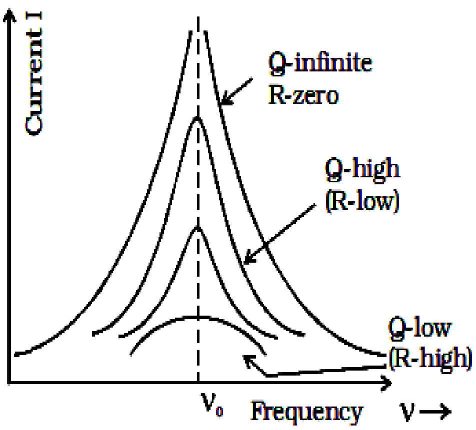
$$X_c = X_L \text{ or } \frac{1}{\omega_0 C} = \omega_0 L \qquad \omega_0 = 2\pi\nu_0 = \frac{1}{\sqrt{LC}}$$

$$\text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \cdots\cdots(1) \qquad \nu_0 = \frac{1}{2\pi\sqrt{LC}} \quad \cdots\cdots(2)$$

Equation (i) gives angular resonant frequency and (ii) gives the resonant frequency

Resonant curve :- variation of current with frequency





NCERT-XII / Unit- 7 – Alternating Current

Find the expression for Sharpness of resonance / Quality factor of resonance

Let ω_1 and ω_2 be symmetrical about ω_0 , at which , the current amplitude is $1/\sqrt{2}$ times its maximum value of current , such that power dissipated by the circuit becomes half . We may write

$\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is called the *bandwidth* of the circuit.

The quantity $(\omega_0 / 2\Delta\omega)$ is regarded as a measure of the sharpness of resonance.

at $\omega = \omega_1$, $$\frac{i_m}{\sqrt{2}} = \frac{v_m}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} = \frac{i_m R}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

or $$\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = R\sqrt{2}$$

or $$R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2$$

or $$\omega_1 L - \frac{1}{\omega_1 C} = R$$

or $$(\omega_0 + \Delta\omega) L - \frac{1}{(\omega_0 + \Delta\omega) C} = R$$

or $$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{1}{\omega_0 C \left(1 + \frac{\Delta\omega}{\omega_0}\right)} = R$$

or $$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{\omega_0 L}{\left(1 + \frac{\Delta\omega}{\omega_0}\right)} = R \qquad \left(\text{Using } \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}\right)$$

or $$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right)^{-1} = R$$

$$\left[\text{since } \frac{\Delta\omega}{\omega_0} << 1, \quad \left(1 + \frac{\Delta\omega}{\omega_0}\right)^{-1} = \left(1 - \frac{\Delta\omega}{\omega_0}\right)\right]$$

or $$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \omega_0 L \left(1 - \frac{\Delta\omega}{\omega_0}\right) = R$$

or $$\omega_0 L \frac{2\Delta\omega}{\omega_0} = R \Rightarrow \Delta\omega = \frac{R}{2L}$$

The sharpness of resonance is given by,

$$\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

The ratio $\frac{\omega_0 L}{R}$ is also called the *quality factor*, *Q* of the circuit.

$$Q = \frac{\omega_0 L}{R}$$

So, larger the value of *Q*, the smaller is the value of $2\Delta\omega$ or the bandwidth and sharper is the resonance.

If the resonance should be sharp, the bandwidth will be less , the tuning of the circuit will veryt be good. If the resonance is sharp , the selectivity of the circuit is very less .

If quality factor is large, i.e., *R* is low or *L* is large, the circuit is more selective.

Q-factor of resonant circuit

The selectivity or sharpness of a resonant circuit is measured by the quality factor or Q factor.

In other words it refers to the sharpness of tuning at resonance.

The Q factor of a series resonant circuit is defined as the ratio of the voltage across a coil or capacitor at resonance to the voltage across the resistance.

$$Q = \frac{\text{voltage across } L \text{ or } C \text{ at resonance}}{\text{voltage across } R}$$

$$Q = \frac{I\omega_o L}{IR} = \frac{\omega_o L}{R} = \frac{1}{\sqrt{LC}}\frac{L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Applications of Resonant circuits

How resonant circuit is used in the tuning mechanism of a radio or a TV set ?

The signals from different broadcasting stations, picked up by the antenna of a radio acts as a source for the tuning circuit , so that the circuit can be driven at many frequencies. To hear one particular radio station, we tune the radio , by varying the capacitance of the capacitor in the tuning circuit such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received. When this happens, the amplitude of the current with the frequency of the signal of the particular radio station in the circuit is maximum.

What type of circuit exhibits resonance ?

The resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other (both being out of phase) and the current amplitude is vm/R, the total source voltage appearing across R. This means that we cannot have resonance in a RL or RC circuit.

NCERT-XII / Unit- 7 – Alternating Current

POWER IN PURELY RESISTIVE CIRCUIT

We know the emf in purely resistive circuit is in phase with current . They are given below as

$e = E_0 \sin \omega t$ ……………….….1

and $i = I_0 \sin \omega t$ ……………..2

The instantaneous value of power is given as

$P = dW / dt \Rightarrow dW = eidt = E_0 I_0 \sin^2 \omega t\, dt$

$= E_0 I_0 [\, ½(1 – \cos 2\omega t)\, dt\,]$

$= ½E_0 I_0 [dt – \cos 2\omega t\, dt\,]$ ………….3

So total work done during the whole cycle of the given ac

$W = {}_0\int^W dW = ½E_0 I_0 [\, {}_0\int^T dt - {}_0\int^T \cos 2\omega t\, dt\,]$

$= ½E_0 I_0 [\, \{t\}_0^T - \{\sin 2\omega t /2\omega \}_0^T\,]$

$= ½E_0 I_0 [\{T-0\} - \{\sin 2(2\pi/T)T - \sin 0\}/2\omega]$

$= ½E_0 I_0 [.T - \{ 0 - 0\}/2\omega]$

$\Rightarrow W = (E_0/\sqrt{2})(I_0/\sqrt{2})\ .T$

$\Rightarrow W/T = E_{RMS} I_{RMS}$

$\Rightarrow$ $\mathbf{P = E_{RMS} I_{RMS}}$ **………….4**

NCERT-XII / Unit- 7 – Alternating Current

POWER IN PURELY INDUCTIVE CIRCUIT

We know the current in purely inductive circuit is lagging behind emf by a phase angle of $\pi/2$. They are given below as

$e = E_0 \sin \omega t$ ……………………….1

and $i = I_0 \sin(\omega t - \pi/2)$ ……….2

The instantaneous value of power is given as

$P = dW / dt \Rightarrow dW = eidt = E_0 I_0 \sin \omega t \sin(\omega t - \pi/2) dt$

$= - E_0 I_0 [\sin \omega t \cos \omega t \, dt]$

$= - E_0 I_0 [\frac{1}{2}(2 \sin \omega t \cos \omega t) dt]$

$= - \frac{1}{2} E_0 I_0 [\sin 2\omega t \, dt]$ ………….3

So total work done during the whole cycle of given ac

$W = {}_0\int^W dW = - \frac{1}{2} E_0 I_0 [{}_0\int^T \sin 2\omega t \, dt]$

$= - \frac{1}{2} E_0 I_0 [- \cos 2\omega t / 2\omega]_0^T$

$= \frac{1}{2} E_0 I_0 [\cos 2(2\pi/T)T - \cos 0] / 2\omega]$

$= \frac{1}{2} E_0 I_0 [1 - 1] / 2\omega$

$\Rightarrow W = 0$

$\Rightarrow$ **$P = 0$ 4**

- ⇨ Current in purely inductive circuit is called wattless current as no power is consumed

NCERT-XII / Unit- 7 – Alternating Current

POWER IN PURELY CAPACITIVE CIRCUIT

We know the current in purely inductive circuit is ahead of emf by a phase angle of $\pi/2$.

They are given below as

$e = E_0 \sin \omega t$ ……………………….1

and $i = I_0 \sin (\omega t + \pi/2)$ ……….2

The instantaneous value of power is given as

$P = dW / dt \Rightarrow dW = eidt = E_0I_0 \sin \omega t \sin (\omega t + \pi/2)dt$

$= E_0 I_0 [\sin \omega t \cos \omega t \, dt]$

$= E_0 I_0 [½(2 \sin \omega t \cos \omega t) dt]$

$= ½E_0 I_0 [\sin 2\omega t \, dt]$ …………3

So total work done during the whole cycle of given ac

$W = {}_0\int^W dW = ½E_0 I_0 [{}_0\int^T \sin 2\omega t \, dt]$

$= ½E_0 I_0 [- \cos2\omega t /2\omega]_0^T$

$= - ½E_0 I_0 [\cos2(2\pi/T)T – \cos 0] /2\omega]$

$= - ½E_0 I_0 [1– 1] / 2\omega$

$\Rightarrow W = 0 \quad \Rightarrow$ **P = 0 …………………..4**

⇨ Current in purely capacitive circuit is called wattless current as no power is consumed

NCERT-XII / Unit- 7 – Alternating Current

POWER IN LCR CIRCUIT

Let the emf in LCR circuit is ahead of current by a phase angle of ϕ as given below

$e = E_0 \sin(\omega t + \phi)$ 1

and $i = I_0 \sin \omega t$ 2

The instantaneous value of power is given as

$P = dW / dt \Rightarrow dW = ei\,dt = E_0 I_0 \sin \omega t \sin(\omega t + \phi)dt$

$= E_0 I_0 [\sin^2\omega t \cos \phi dt + \sin \omega t \cos \omega t \sin\phi\, dt]$

$= E_0 I_0 [\frac{1}{2}(1 - \cos 2\omega t) \cos \phi dt + \frac{1}{2}(2 \sin\omega t \cos \omega t)\sin\phi dt]$

$= \frac{1}{2} E_0 I_0 [\cos \phi dt - \cos 2\omega t \cos \phi dt + \sin 2\omega t \sin \phi\, dt]$...3

So total work done during the whole cycle of he given ac

$W = {}_0\int^W dW = \frac{1}{2} E_0 I_0 [\cos \phi \,{}_0\int^T dt - \cos \phi\, {}_0\int^T \cos 2\omega t\, dt + \sin \phi\, {}_0\int^T \sin 2\omega t\, dt]$

$= \frac{1}{2} E_0 I_0 [\cos \phi \{t\}_0^T - \cos \phi \{\sin 2\omega t / 2\omega\}_0^T + \sin \phi \{-\cos 2\omega t / 2\omega\}_0^T]$

$= \frac{1}{2} E_0 I_0 [\cos \phi\{T-0\} - \cos \phi\{\sin 2(2\pi/T)T - \sin 0\}/2\omega - \sin\phi\{\cos 2(2\pi/T) T - \cos 0\}/2\omega]$

$= \frac{1}{2} E_0 I_0 [\cos \phi .T - \cos \phi\{0 - 0\}/2\omega - \sin\phi\{1 - 1\}/2\omega]$

$\Rightarrow W = (E_0/\sqrt{2})(I_0/\sqrt{2}) \cos \phi .T$

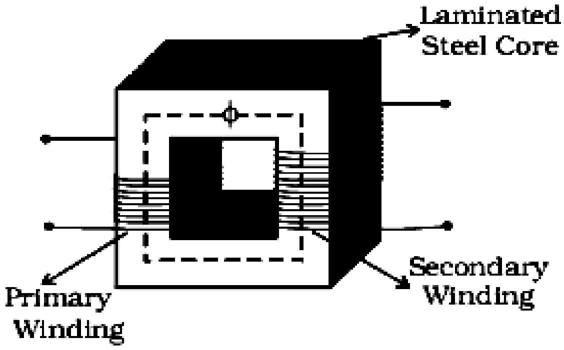
$\Rightarrow W/T = E_{RMS} I_{RMS} \cos \phi$.

$\Rightarrow P = E_{RMS} I_{RMS} \cos \phi$ 4

⇨ The term $\cos \phi$ is known as power factor of LCR circuit ,
which is equal to $\cos \phi = R / Z = R / (X_L - X_C)$

NCERT-XII / Unit- 7 – Alternating Current

Transformer



Transformer is an electrical device used for converting low alternating voltage into high alternating voltage and vice versa and transfering electric power from one circuit to another.

It is based on the principle of electromagnetic induction.

A transformer consists of primary and secondary coils insulated from each other, wound on a soft laminated iron core to minimise eddy currents . The a.c. input is applied across the primary coil ,which produces a varying magnetic flux in it. This, in turn , produces a varying magnetic flux in the secondary. Hence, an induced emf is produced across the secondary.

Let E_p and E_s be the induced emf in the primary and secondary coils and N_P and N_s be the number of turns in primary and secondary coils respectively. Since same flux links with the primary and secondary, the emf induced per turn of the two coils must be the same

$$\frac{E_P}{N_P}=\frac{E_S}{N_S} \quad \text{or,} \quad \frac{E_P}{E_S}=\frac{N_P}{N_S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For an ideal transformer,

input power = output power = > $E_P I_P = E_S I_S$,

here I_P & I_S are currents in primary & secondary coils.

$$\text{(i.e.)} \quad \frac{E_s}{E_p}=\frac{I_P}{I_s} \qquad \ldots(2)$$

From equations (1) and (2) $\dfrac{E_s}{E_p}=\dfrac{N_s}{N_p}=\dfrac{I_P}{I_S}= k$

where k is called transformer ratio.

for step up transformer k >1 & for step down transformer k <1

In step up transformer , $E_S>E_P$ implying that $I_S<I_P$.

Thus a step up transformer increases the voltage by decreasing the current, which is in accordance with the law of conservation of energy. Similarly a step down transformer decreases the voltage by increasing the current.

EFFICIENCY:

$$\eta = \frac{output\ \ power}{input\ \ power} = \frac{E_s I_s}{E_p I_p}$$

The efficiency η = 1 (i.e. 100%), only for an ideal transformer where there is no power loss. But practically there arc numerous factors leading to energy loss in a transformer and hence the efficiency Is always less than 1

Energy losses in a transformer

(1) Hysteresis loss : The repeated magnetisation and demagnetisation of the iron core caused by the alternating Input current, produces loss in energy called hystcrisis loss. This loss can be minimised by using a core with a material having the least hysteresis loss. Alloys like mumetal and silicon steel arc used to reduce hvsterisis loss.

(2) Copper loss: The current flowing through the primary and secondary windings lead to Joule heating effect. Hence some energy is lost in the form of heat. Thick wires with considerably low resistance are used to minimise this loss.

(3) Eddy current loss (Iron loss):The varying magnetic flux produces eddy current In the core. This leads to the wastage of energy in the form of heat. This loss Is minimised by using a laminated core made of stelloy. an alloy of steel.

(4) Flux loss: The flux produced In the primary coil is not completely linked with the secondary coil due to leakage. This results in the loss of energy. Tills loss can be minimised by using a shell type core.

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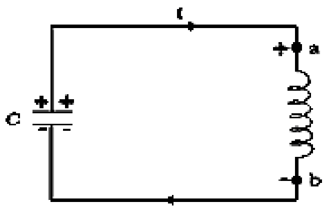
(5) In addition to the above losses, due to the vibration of the core, sound is produced, which causes a loss in the energy.

LC OSCILLATIONS



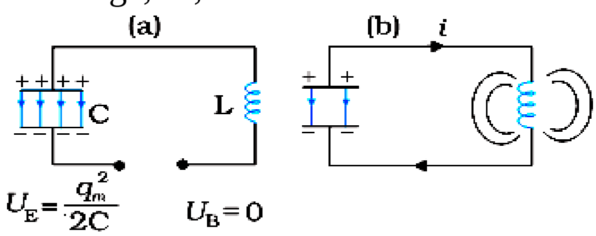
We know that a capacitor and an inductor can store electrical and magnetic energy, respectively. When a capacitor (initially charged) is connected to an inductor, the charge on the capacitor and the current in the circuit exhibit the phenomenon of electrical oscillations.

Let a capacitor be charged q_m (at t = 0) and connected to an inductor . The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to current in the circuit.

According to Kirchhoff's loop rule,

$\frac{q}{C} - L\frac{di}{dt} = 0$ => $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$ [as i = – (dq/dt)]

The charge,so , oscillates with a natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$



At t = 0, the switch is closed and the capacitor starts to discharge [Fig.(b)]. As the current increases, it sets up a magnetic field in the inductor and thereby, some

energy gets stored in the inductor in the form of magnetic energy: $U_B = ½ Li^2$

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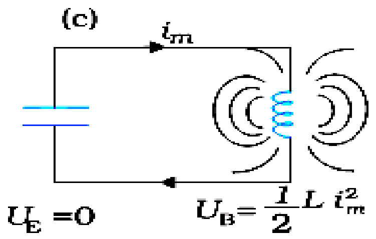
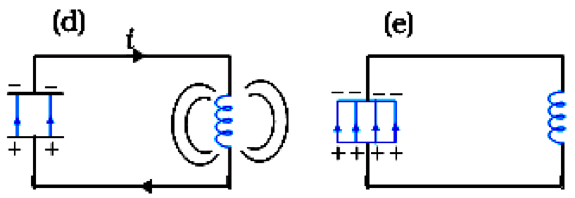


As the current reaches its maximum value i_m, (at t = T/4) as in Fig.(c), all the energy is stored in the magnetic field: $U_B = ½ L\, i_m^{\,2}$. We can easily check that the maximum electrical energy equals the maximum magnetic energy. The capacitor now has no charge and hence no energy.



The current now starts charging the capacitor, as in Fig. (d). This process continues till the capacitor is fully charged (at t = T/2) [Fig. (e)]. But it is charged with a polarity opposite to its initial state in Fig. (a).

The whole process just described will now repeat itself till the system reverts to its original state.

Thus, the energy in the system oscillates between the capacitor and the inductor.

<u>LC oscillations is not realistic for two reasons:</u>

(i)Every inductor has some resistance. The effect of this resistance is to introduce a damping effect on the charge and current in the circuit and the oscillations finally die away.

(ii) Even if the resistance were zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves . In fact, radio and TV transmitters depend on this radiation.