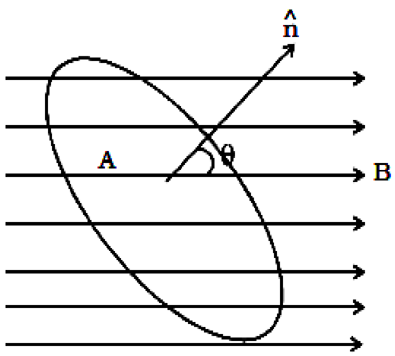
Tridib's Physics Tutorials

NCERT-XII / Unit- 6 – Electromagnetic Induction

Electromagnetic Induction-

The phenomenon in which electric current is generated by varying magnetic fields is appropriately called electromagnetic induction.

Magnetic .flux.



The magnetic **flux** (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing a closed area (A) . If θ is the angle between the direction of the field and normal to the area. Then

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

SI unit of magnetic flux : **Weber**

Faraday's laws of electromagnetic induction

First law:-Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second law:- The magnitude of emf induced in a closed circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

Expression for induced emf

If dφ is the change in magnetic flux in a time dt,then mathematically , induced emf is

$$e \propto \frac{d\Phi_B}{dt} \Rightarrow e = k\frac{d\Phi_B}{dt} \Rightarrow e = -\frac{d\Phi_B}{dt}$$ (According to Lenz , k = -1)

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Lenz's law :-

Lenz's law states that " **the induced current produced in a circuit always flows in such a direction that it opposes the change or cause that produces it."**

Prove that " Lenz's law obeys the principle of consequence of conservation of energy."

When the north pole of magnet is moved towards a coil , the face of the coil facing the magnet acquires north polarity .so work has to be done by the magnet to move it further against the force of repulsion at cost of mechanical energy stored in it , which will appear as the electrical energy of the coil .

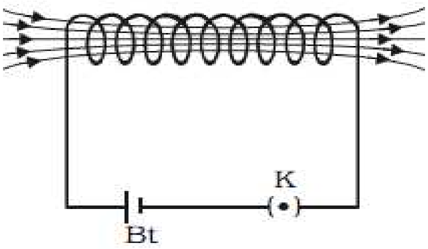
When the magnet is moving away from the coil , the same face acquires south polarity. Now the work done is done by the coil at cost of electrical energy store in it , which will reappear in the magnet as its mechanical energy .

Thus conversion of energy from one form to another and vce versa confirms that Lenz's law is the consequence of conservation of energy.

Self Induction

The property of a coil which enables to produce an opposing induced emf in it when the current in the coil changes is called self induction

Expression for Coefficient of self induction.



When a current I flows through a coil, the magnetic flux ϕ linked with the coil is proportional to the current.

$\phi \propto I$ or $\phi = LI$

where L is a constant of proportionality and Is called coefficient of self induction or self inductance.

If I = 1A. ϕ = L x l. then L = ϕ , Therefore, coefficient of self induction of a coil is numerically equal to the magnetic flux linked with a coil when unit current flows through it. According to laws of electromagnetic induction.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) \quad \text{or} \quad e = -L\frac{dI}{dt}$$

$$\text{If } \frac{dI}{dt} = 1\ \text{A s}^{-1}, \text{ then } L = -e$$

The coefficient of self induction of a coil is numerically equal to the opposing emf induced in the coil when the rate of change of current jh rough the coil is unity. The unit of self Inductance is henry (H). ,

One henry is defined as the self-inductance of a coil in which a change in current of one ampere per second produces an opposing emf of one volt .

Self inductance of a long solenoid

Let us consider a solenoid of N turns with length ***l*** and area of cross section A. It carries a current I. If B is the magnetic field at any point inside the solenoid, then

Magnetic flux per turn = B x area of each turn

$$B_1 = \mu_0 \frac{N}{l} I \ \ldots\ldots\ldots\ldots\ldots(1)$$

Magnetic flux per turn , $\phi = \mu_0 \frac{N}{l} IA \ \ldots\ldots\ldots\ldots(2)$

Hence, the total magnetic flux (ϕ) linked with the solenoid is given by the product of flux through each turn and the total number of turns.

$$\Phi = \mu_0 \frac{N}{l} IAN = \mu_0 \frac{N^2}{l} IA \ \ldots..(3)$$

If L is the coefficient of self induction of the solenoid, then

$$\phi = LI \ \ldots(4)$$

From equations (3) and (4), => $LI = \mu_0 \frac{N^2 IA}{l}$ => $L = \frac{\mu_0 N^2 A}{l}$ ….(5)

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Energy associated with an inductor

Whatever current flows through a coil, the self-inductance opposes the growth of the current. Hence, some work has to be done by external agencies in establishing the current. If c is the induced cmf then.

$$e = -L\frac{di}{dt} \quad ....(1)$$

The small amount of work dw done in a time Interval dt is

$$dW = eIdt = -L\frac{di}{dt}.Idt = -LIdi \quad ....(2)$$

The total work done when the current Increases from 0 to maximum value I_0 is

$$W = \int dW = \int_0^{I_0} -LIdi = -L\int_0^{I_0} Idi = -\frac{1}{2}L[I^2]_0^{I_0} = -\frac{1}{2}LI_0^{\ 2} \quad ....(3)$$

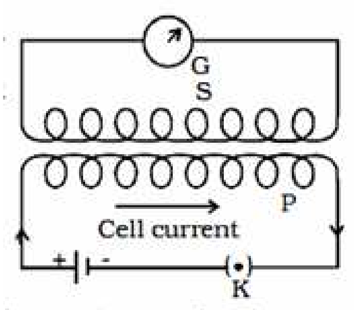
Tills work done is stored as magnetic potential energy in the coll.

Energy stored in the coil $U = W = -\frac{1}{2}LI_0^{\ 2}$ ....(4)

Negative sign is consequence of Lcnz's Law.

Hence. quantitatively, the energy stored in an inductor Is $U = \frac{1}{2}LI_0^{\ 2}$ ....(5)

Mutual induction



The phenomenon of producing an induced emf In a coil due to the change in current in the other coil is known as mutual induction.

Coefficient *of mutual induction*

P and S are two coils placed close to each other p is connected to a battery S is connected to a galvanometer G. I_p Is the current In coil P and ϕ_S Is the magnetic flux linked with coil S due to the current in coil P.

$\phi_S \propto I_P$ or $\phi_S = M I_p$

where M is a constant of proportionality and is called the coefficient of mutual induction or mutual inductance between the two coils

If I_p = 1A. then. $M = \phi_S$

Thus, coefficient of mutual induction of two colls is numerically equal to the magnetic flux linked with one coll when unit current flows through the neighbouring coil. If e Is the induced emf in the coil (S) at any Instant of time, then from the laws of electromagnetic Induction.

$$e_s = -\frac{d\phi_s}{dt} = -\frac{d}{dt}(MI_P) = -M\frac{dI_P}{dt}$$

$$\therefore M = -\frac{e_s}{\left(\frac{dI_P}{dt}\right)} \qquad \text{If } \frac{dI_P}{dt} = 1\ \text{A s}^{-1}, \text{ then, } M = -e_s$$

Thus, the coefficient of mutual Induction of two coils is numerically equal to the em!" induced in one coil when the rate of change of current through the other coil is unity. The unit of coefficient of mutual induction is henry.

One henry is defined as the coefficient of mutual induction between a pair of coils when a change of current of one ampere per second in one coil produces an induced emf of one volt in the other coil.

The coefficient of mutual induction between a pair of coils depends on the following factors

(i) Size and shape of the coils, number of turns and permeability of material on which the coils are wound.

(ii) proximity of the coils

Mutual induction of two long coaxial solenoids.

S_1 and $\mathbf{S_2}$ are two long solenoids each of length $\boldsymbol{l}$. The solenoid S_2 is wound closely over the solenoid S_1, N_1 and N_2 arc the number of turns in the solenoids S_1 and S_2 respectively. Both the solenoids are considered to have the same area of cross section A as they are closely wound together. I_1 is the current flowing through the solenoid S_1. The magnetic field B_1 produced at any point inside the solenoid S, due to the current I_1 is

$$B_1 = \mu_0 \frac{N_1}{l} I_1 \text{ ....(1)}$$

The magnetic flux linked with each turn of S_2 is equal to B_1A. Total magnetic flux linked with solenoid S_2 having N_2 turns is

$\phi_2 = B_1AN_2$.......................(2)

Substituting for B_1 from equation (1)

$$\phi_2 = (\mu_0 \frac{N_1}{l} I_1)AN_2 = \frac{\mu_0 N_1 N_2 A I_1}{l} \text{ ....(3)}$$

But $\phi_2 = M I_1$ ....(4)

where M is the coefficient of mutual Induction between S_1 and S_2.

From equations (3) and (4) ,

$$MI_1 = \frac{\mu_0 N_1 N_2 A I_1}{l} \Rightarrow M = \frac{\mu_0 N_1 N_2 A}{l} \text{ ....(5)}$$

If the core is filled with a magnetic material of permeability μ

$$M = \frac{\mu N_1 N_2 A}{l} \text{ ....(6)}$$

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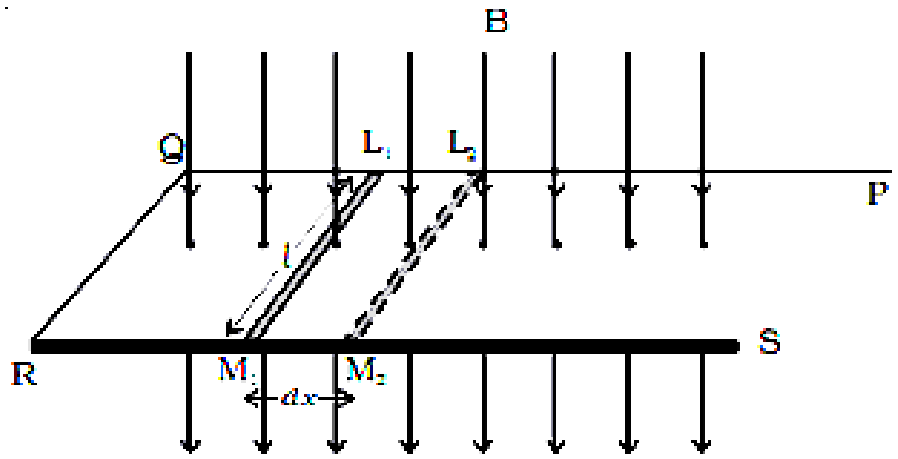
Emf induced by changing the area enclosed by the coil

PQRS is a conductor bent in the shape as shown In the fig L_1M_1 is a sliding conductor of length ***l*** resting on the arms PQ and RS. A uniform magnetic field 'B' acts perpendicular to the plane of the conductor. The closed area of the conductor is L_1QRM_1 . When L_1M. is



moved through a distance distance ***dx*** in time ***dt***. the new area is L_2QRM_2. Due to the change in area $L_2L_1M_1M_2$, there is a change in the flux linked with the conductor . Therefore . an induced emf is produced.

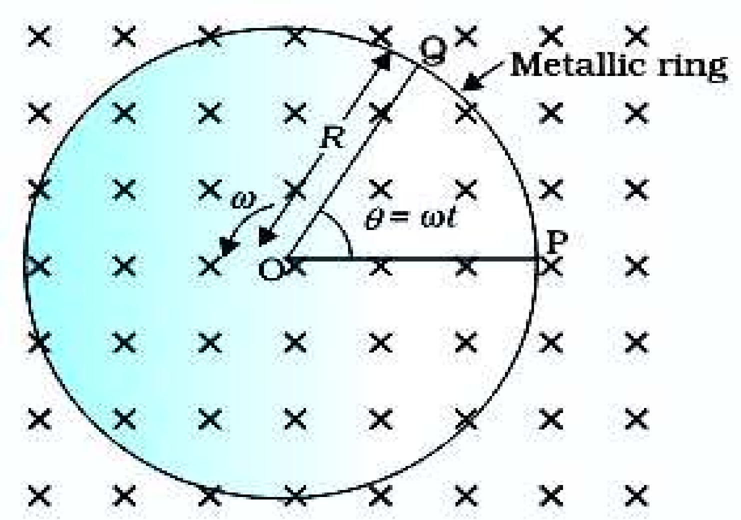
Change in area ***dA*** = Area $L_2L_1M_1M_2$ = ***l dx***

Change in the magnetic flux, $d\phi = B.dA = B\,l\,dx$

But $$e = -\frac{d\phi}{dt} = -\frac{Bldx}{dt} = -Blv$$

where v is the velocity with which the sliding conductor is moved.

Rotational emf



To calculate the emf induced in a rod of radius R ,rotating uniformly in a magnetic field B , we imagine that in a small time interval dt , it changes its position from OP to OQ by making an angle dθ at O .

Area OPQ = ½ X OP X PQ

By definition of angle dθ = arc PQ / radius OP

By Faraday's law of electromagnetic induction , the emf induced in the rod

$e = B \times$ (rate of change of area of loop)

$= B \times (\frac{1}{2} \times OP \times PQ) / dt$

$= B \times (\frac{1}{2} \times OP \times OP \times d\theta) / dt$

$= B \times (\frac{1}{2} \times OP^2 \times d\theta) / dt$

$= \mathbf{B} \times (\frac{1}{2} \times R^2 \times d\theta) / dt$

so , $\mathbf{e = \frac{1}{2} B R^2 \omega}$ …….(3)

Eddy currents or Foucault currents -

When a metallic body moves in a magnetic field or when the magnetic field through a stationary metallic body is altered, induced current is produced in the metal. This current is called eddy current.

The direction of the eddy current is given by Lenz's law.

Disadvantages of Eddy Current

(i) Eddy current involves excessive production of heat , causing loss of energy.

(ii) It creats wear and tear to the life of conductors .

Eddy currents are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material . The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths , reducing the strength of the eddy currents.

Eddy currents are used to advantage in certain applications like:

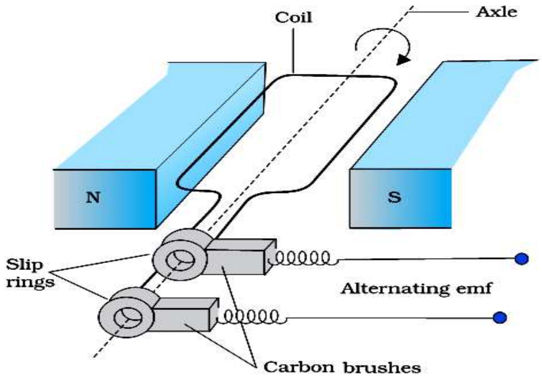
(i) **Magnetic braking in trains**: Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.

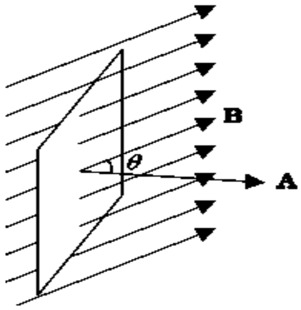
(ii) **Electromagnetic damping**: Certain galvanometers have a fixed core made of nonmagnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

(iii) **Induction furnace**: Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.

(iv) **Electric power meters**: The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

AC GENERATOR





An ac generator converts mechanical energy into electrical energy.

Principle of ac generator:- By changing the loop's orientation or changing its effective area , an emf or current is induced in the loop . As shown in above figure , it consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means.

The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil.

The ends of the coil are connected to an external circuit by means of slip rings and brushes.

When the coil is rotated in the magnetic field B with a constant angular speed $\omega(\omega = \theta/t)$, at any instant t , the effective area of the coil is $A\cos\theta$, where θ is the angle between A and B. So,the magnetic flux at t is $\Phi_B = BA\cos\theta = BA\cos\omega t$

From Faraday's law, the induced emf for the rotating coil of N turns is then,

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t)$$

Thus, the instantaneous value of the emf is $\varepsilon = NBA\,\omega\sin\omega t$ …………(1)

where **NBAω** is the maximum value of the emf = ε_0, which occurs when $\sin\omega t = \pm 1$.

$\boldsymbol{\varepsilon = \varepsilon_0 \sin\omega t}$ ……………….(2)

The emf has its extremum value when $\theta = 90^{\circ}$ or $\theta = 270^{\circ}$, as the change of flux is greatest at these points.

The direction of the current changes periodically and therefore the current is called alternating current (ac). Since $\omega = 2\pi\nu$, Eq (2) can be written as $\boldsymbol{\varepsilon = \varepsilon_0 \sin 2\pi\nu t}$ ……….(3)

where ν is the frequency of revolution of the generator's coil.

Eq. (2) and (3) give the instantaneous value of the emf and ε varies between $+\varepsilon_0$ and $-\varepsilon_0$ periodically.

Types of commercial generators

Hydro-electric generators, the mechanical energy required for rotation of the armature is provided by water falling from a height, i.e, from dams. .

Thermal generators water is heated to produce steam using coal or other sources. The steam at high pressure produces the rotation of the armature.

Nuclear power generators. Instead of coal, a nuclear fuel can also be used,

The frequency of rotation is 50 Hz in India.

In certain countries such as USA, it is 60 Hz.