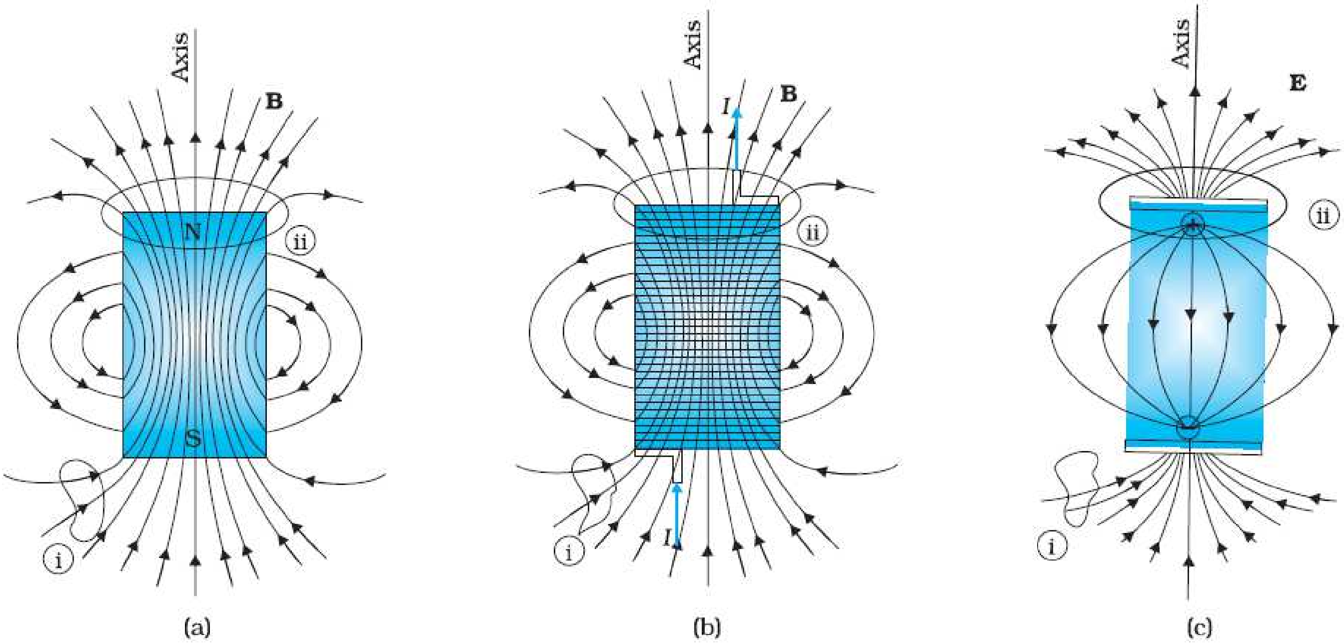
NCERT-XII / Unit- 5 – Magnetism and matter

The magnetic field lines

The magnetic field lines are a visual and intuitive realisation of the magnetic field. Their properties are:

(i)The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii)The tangent to the field line at a given point represents the direction of the net magnetic field B at that point.



(a) The field lines of a bar magnet, (b) a current-carrying finite solenoid , (c) electric dipole. At large distances, the field lines are very similar. The curves labelled i and ii are closed Gaussian surfaces.

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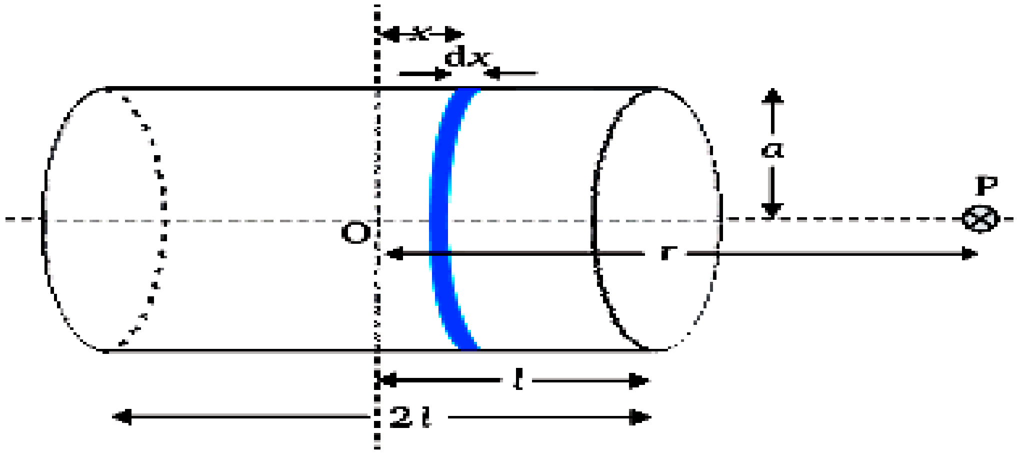
Bar magnet as an equivalent solenoid



Let us consider a solenoid of length **2*l***, radius a and carrying a current I and having n turns per unit length. Considering a point P at a distance r from the centre O of solenoid. Considering an element of solenoid of length dx at a distance x from its centre. This element is a circular current loop having (ndx) turns. The magnetic field at axial point P due to this current loop is

$$dB = \frac{\mu_0 n\, dx\, I a^2}{2[(r-x)^2 + a^2]^{3/2}} \qquad \text{……………………..…….1}$$

The magnitude of the total field is obtained by summing over all the elements — in other words by integrating from x = – l to x = + l . Thus,

$$B = \frac{\mu_0 n I a^2}{2}\int_{-l}^{l}\frac{dx}{[(r-x)^2 + a^2]^{3/2}} \qquad \text{…………………..2}$$

Considering the far axial field of the solenoid, i.e., r >> a and r >> l .So the denominator is approximated by

$$[(r-x)^2 + a^2]^{3/2} \approx r^3$$

$$\text{and } B = \frac{\mu_0 n I a^2}{2r^3}\int_{-l}^{l} dx = \frac{\mu_0 n I}{2}\,\frac{2l a^2}{r^3} \qquad \text{…….3}$$

The magnitude of the magnetic moment of the solenoid is,

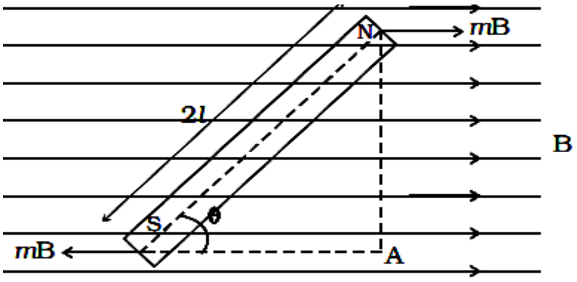
m = total number of turns × current × cross-sectional area = n (2l) x I x (πa^2) .

Thus, $B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$ …………………………..4

This is also the far axial magnetic field of a bar magnet. Thus, a bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

So a bar magnet as an equivalent solenoid

Torque on a bar magnet placed in a uniform magnetic field



Considering a bar magnet NS of length 2l and pole strength m placed in a uniform magnetic field of induction B at an angle θ with the direction of the field .

Due to the magnetic field B, a force mB acts on the north pole along the direction of the field and a force mB acts on the south pole along the direction opposite to the magnetic field. These two forces are equal and opposite, hence constitute a couple. The torque τ due to the couple is

τ = one of the forces × perpendicular distance bet them

τ = F × NA = mB × NA = mB × 2l sin θ

or **τ = MB sin θ** ……………....(2)

Vectorially, $\vec{\tau} = \vec{M} \times \vec{B}$, The direction of **τ** is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$

If B = 1 and θ = 90° Then from equation (2), τ = M Hence, moment of the magnet M is equal to the torque necessary to keep the magnet at right angles to a magnetic field of unit magnetic induction.

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A freely suspending bar magnet execute SHM . Find its time period . How magnetic field by using a bar magnet is calculated?

To determine the magnitude of B accurately by a small compass needle of known magnetic moment m and moment of inertia $\mathcal{I}$ and allowing it to oscillate in the magnetic field.

The torque on the needle is $\vec{\tau} = \vec{m} \times \vec{B}$……….1

In magnitude **τ = mB sinθ** , Here τ is restoring torque and θ is the angle between m & B.

Therefore, in equilibrium $\mathcal{I}\frac{d^2\theta}{dt^2} = -mB\sin\theta$ ……….2

Negative sign with mB sinθ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate sin θ ≈ θ and get

$\mathcal{I}\frac{d^2\theta}{dt^2} = -mB\theta$ or, $\frac{d^2\theta}{dt^2} = -\frac{mB}{\mathcal{I}}\theta$ ……………3

This represents a simple harmonic motion.

The square of the angular frequency is $\omega^2 = mB/\mathcal{I}$ and the time period is,

$T = 2\pi\sqrt{\frac{\mathcal{I}}{mB}}$ or $B = \frac{4\pi^2\mathcal{I}}{mT^2}$

Knowing time period of oscillation of a compass needle and its moment of inertia B can be calculated

Magnetic potential energy

Let θ be the angle between magnetic moment m of a bar magnet and magnetic field B , where it is placed and experiencing a torque of magnitude **τ = mB sinθ ……1**

Let it be rotated by a small angle dθ , required workdone is **dW= τ dθ = mB sinθ dθ 2**

So total workdone , which will be stored in the bar magnet as its magnetic potential energy is

$U_m = W = \int dW = \int mB\sin\theta\, d\theta = -mB\cos\theta$ ……3

In vector form , $U_m = \vec{m}\cdot\vec{B}$……………4

NCERT-XII / Unit- 5 – Magnetism and matter

GAUSS'S LAW in magnetism:- **"The net magnetic flux is zero for both the surfaces. "**

Mathematically $$\phi_B = \sum_{all} \Delta\phi_B = \sum_{all} \mathbf{B}\cdot\Delta\mathbf{S} = 0$$

The difference between the Gauss's law of magnetism and that for electrostatics is that isolated magnetic poles (also called monopoles) are not known to exist.

The simplest magnetic element is a dipole or a current loop. All magnetic phenomena can be explained in terms of an arrangement of dipoles and/or current loops.

Current loop as a magnetic dipole

Ampere found that the distribution of magnetic lines of force around a finite current carrying solenoid is similar to that produced by a bar magnet. This is evident from the fact that a compass needle when moved around these two bodies show similar deflections. After noting the close resemblance between these two, Ampere demonstrated that a simple current loop behaves like a bar magnet and put forward that all the magnetic phenomena is due to circulating electric current. This is Ampere's hypothesis.

The magnetic induction at a point along the axis of a circular coil carrying current is

$$B = \frac{\mu_o n I a^2}{2\,(a^2+x^2)^{\frac{3}{2}}}$$

The direction of this magnetic field is along the axis and is given by right hand rule. For points which are far away from the centre of the coil, x>>a, a^2 is small and it is neglected. Hence for such points,

$$B = \frac{\mu_o n I a^2}{2x^3}$$

If we consider a circular loop, n = 1, its area A = πa^2

$$\therefore \qquad B = \frac{\mu_o I A}{2\pi x^3} \qquad \text{...(1)}$$

The magnetic induction at a point along the axial line of a short bar magnet is

$$B = \frac{\mu_o}{4\pi} \cdot \frac{2M}{x^3} \quad \text{or} \quad B = \frac{\mu_o}{2\pi} \cdot \frac{M}{x^3} \qquad ...(2)$$

Comparing equations (1) and (2), we find that

$$M = IA \qquad ...(3)$$

Hence a current loop is equivalent to a magnetic dipole of moment M = IA

The magnetic moment of a current loop is defined as the product of the current and the loop area. Its direction is perpendicular to the plane of the loop.

,,,

The magnetic dipole moment of a revolving electron

According to Neil Bohr's atom model, the negatively charged electron is revolving around a positively charged nucleus in a circular orbit of radius r. The revolving electron in a closed path constitutes an electric current. The motion of the electron in anticlockwise direction produces conventional current in clockwise direction.

Current, i = e / T , where T is the period of revolution of the electron.

If v is the orbital velocity of the electron, then $T = \frac{2\pi r}{v}$ $\therefore$ $i = \frac{ev}{2\pi r}$

Due to the orbital motion of the electron , there will be orbital magnetic moment μ_l

μ_l = iA, where A is the area of the orbit

$$\mu_l = \frac{ev}{2\pi r} \cdot \pi r^2 \Rightarrow \mu_l = \frac{evr}{2}$$

If m be the mass of an election , $\mu_l = \frac{e}{2m}(m\,vr) = \frac{e}{2m} l$, mvr is the angular momentum of the electron about nucleus .

Vectorially, $\vec{\mu_l} = -\frac{e}{2m}\vec{l}$

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The ratio $\frac{\mu_l}{l} = \frac{e}{2m_e}$ is called the gyromagnetic ratio & is a constant. It is **8.8 × 10^{10} C /kg** for an electron

According to Bohr's postulate ,

$mvr = \frac{nh}{2\pi}$ where n is a natural number and h is the Planck's constant = 6.626×10^{-34} Js.

$$\boxed{\mu_l = \frac{e}{2m} \cdot \frac{nh}{2\pi} = \frac{neh}{4\pi m}}$$

The minimum value of magnetic moment is

$$(\mu_l)_{min} = \frac{eh}{4\pi m}, \; n = 1$$

The value of $\frac{eh}{4\pi m}$ is called Bohr magneton

By substituting the values of e, h and m, the value of Bohr magneton is found to be 9.27×10^{-24} Am^2

In addition to the magnetic moment due to its orbital motion, the electron possesses magnetic moment due to its spin. Hence the resultant magnetic moment of an electron is the vector sum of its orbital magnetic moment and its spin magnetic moment.

Comparisoin of Ectrostatics and Magnatism

	$1/\varepsilon_0$	μ_0
Dipole moment	$\mathbf{p}$	$\mathbf{m}$
Equatorial Field for a short dipole	$-\mathbf{p}/4\pi\varepsilon_0 r^3$	$-\mu_0\,\mathbf{m}/4\pi r^3$
Axial Field for a short dipole	$2\mathbf{p}/4\pi\varepsilon_0 r^3$	$\mu_0\,2\mathbf{m}/4\pi r^3$
External Field: torque	$\mathbf{p} \times \mathbf{E}$	$\mathbf{m} \times \mathbf{B}$
External Field: Energy	$-\mathbf{p}\cdot\mathbf{E}$	$-\mathbf{m}\cdot\mathbf{B}$

THE EARTH'S MAGNETISM

The strength of the earth's magnetic field varies from place to place on the earth's surface ;its value being of the order of 10^{-5} T.

The vertical plane containing the longitude circle and the axis of rotation of the earth is called the geographic meridian.

The magnetic meridian of a place can be defined as the vertical plane which passes through the imaginary line joining the magnetic north and the south poles

Dynamo effect The magnetic field of earth arises due to electrical currents produced by convective motion of metallic fluids , which is consisting mostly of molten iron and nickel, in the outer core of the earth. This is known as the dynamo effect.

The axis of the magnetic dipole of earth does not coincide with the axis of rotation of the earth but is presently titled by approximately 11.3º with respect to the later.

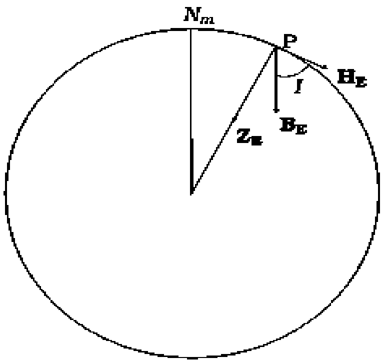
Magnetic elements of earth

The Earth's magnetic field at any point on the Earth can be completely defined in terms of certain physical quantities called magnetic elements of the Earth . There are

(i) Declination or the magnetic variation θ.:- The angle between the true geographic north and the magnetic south of earth is called the magnetic declination or declination

(ii) Dip or inclination δ :- The angle made by the direction of earth's magnetic field with the horizontal component of the Earth's magnetic field is known as angle of dip .

(iii)Horizontal component of the Earth's magnetic field B_E :- It is the horizontal component of earth's magnetic field .



The total magnetic field at P can be resolved into a horizontal component H_E and a vertical component Z_E. The angle that B_E makes with H_E is the angle of dip, I. Representing the verticle component by Z_E, we have

$Z_E = B_E \sin I$ and $H_E = B_E \cos I$

which gives, $\tan I = \dfrac{Z_E}{H_E}$ and $B_E = \sqrt{Z_E^2 + H_E^2}$

MAGNETISATION AND MAGNETIC INTENSITY

We define *magnetisation* M of a sample to be equal to its net magnetic moment per unit volume: $M = \dfrac{m_{net}}{V}$

Dimensional formula of M is $[M^0L^{-1}T^0A^1]$ and SI units of A m^{-1}.

Relation among χ, μ r and μ

Let us consider a long solenoid of *n* turns per unit length and carrying a current *I* & the magnetic field in its interior is

$\mathbf{B_0 = \mu_0 nI}$ ....(1)

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than B_0. The net B field in its interior of the solenoid may be is

$\mathbf{B = B_0 + B_m}$ ....(2)

where B_m is the field due to the material core which. is proportional to the magnetisation M of the material and is expressed

as $\mathbf{B_m = \mu_0 M}$ ....(3) where μ_0 is the permeability of vacuum.

Let us introduce another vector field H, called the *magnetic intensity*, which is defined by

$\mathbf{H} = \dfrac{\mathbf{B}}{\mu_0} - \mathbf{M}$ ......(4)where H has the same dimensions as M

Thus, the total magnetic field B is written a $\mathbf{B = \mu_0(H + M)}$...(5)

So total magnetic field inside the solenoid c ontributed due to two parts:-

(i) due to external factors like current in the solenoid,represented by H.

(ii) due to the specific nature of the magnetic material, represented by M.

The latter quantity can be influenced by external factors ,the influence is mathematically expressed as

$M= \chi H$ ....(6)

where χ ,is a dimensionless quantity ,called the *magnetic susceptibility* , χ is a measure of how a magnetic material responds to an external field

From Eqs. (5) and (6) we obtain,

$B=\mu_0(1+\chi)H = \mu_0\ \mu_r\ H = \mu\ H$ ....(7)

where **$\mu_r= 1 + \chi$**, is a dimensionless quantity called the *relative magnetic permeability* of the substance, which is analogous to the dielectric constant in electrostatics.

The *magnetic permeability* of the substance is μ and it has the same dimensions and units as μ_0

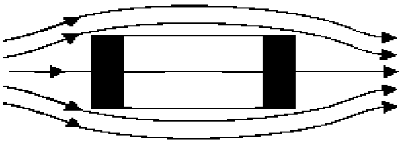
$\mu = \mu_0\mu_r = \mu_0\ (1+\chi)$.

The is the relation among **χ, μ_r and μ** .

1 Diamagnetism

1:Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field.

2:Diamagnetic substance is repelled by a magnet.



3:If a bar of diamagnetic material placed in an external magnetic field ,the field lines are repelled or expelled and the field inside the material is reduced , reduction is slight, equal to one part in 10^5.

4:When placed in a non-uniform magnetic field, the bar will tend to move from high to low field.

5:The value of the susceptibility is negative and very small **$(-1\le \chi < 0)$**

6: Its relative permeability is positive , but less than 1**$(0 \le \mu_r < 1)$**

7: Its permeability is less than absolute permeability, **$(\mu < \mu_0)$**

8: Some diamagnetic materials are bismuth, copper, lead, silicon, nitrogen (at STP), water and sodium chloride. Diamagnetism is present in all the substances. However, the effect is so weak , so it gets shifted by paramagnetism, ferromagnetism, etc.

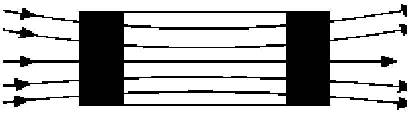
The explanation for diamagnetism Electrons in an atom orbiting around nucleus possess orbital angular momentum .These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up. This happens due to induced current in accordance with Lenz's law . Thus, the substance develops a net magnetic moment in direction opposite to that of the applied field and hence repulsion.

Superconductors These are most exotic diamagnetic materials. These They are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled . $\chi = -1$ **and** $\mu_r = 0$. A superconductor repels a magnet and repelled by the magnet. The phenomenon of perfect diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer. Superconducting magnets can be used for running magnetically levitated superfast trains.

Paramagnetism

1:Paramagnetic substances are those substances , which have tendency to move from a region of weak magnetic field to strong magnetic field .

2: Paramagnetic substances get weakly attracted to a magnet and get weakly magnetised when placed in an external magnetic field.



3:If a bar of paramagnetic material placed in an external field ,the field lines gets concentrated inside the material, and the field inside is enhanced , enhancement is slight, equal to one part in 10^5

4:When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong.

5: The value of the susceptibility is positive $(0 < \chi < \varepsilon)$

6: Its relative permeability is positive , but just greater than 1$(0 < \mu_r < 1)$

7: Its permeability is grater than absolute permeability, $(\mu > \mu_0)$

8: Some paramagnetic materials are aluminium, sodium, calcium ,oxygen (at STP) and copper chloride.

The explanation for paramagnetism

The individual atoms of a paramagnetic material possess a permanent magnetic dipole moment of their own .Due to the ceaseless random thermal motion of the atoms, no net magnetization is seen. In the presence of a very strong external field B_0 and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0.

According to Curie's law, magnetisation of a paramagnetic material is inversely proportional to the absolute temp T,

i.e. $M = C\frac{B_0}{T}$ Or eventually $\chi = C\frac{\mu_0}{T}$ (C is called Curie's constant).

Thus, for a paramagnetic material both χ and μ_r depend not only on the material, but also on the sample temperature. When the temperature is lowered , the field or magnetization is increased until it reaches the saturation value M_s, at which point all the dipoles are perfectly aligned with the field and the substance develops a net magnetic moment in direction of the applied field and hence repulsion.Beyond this, Curie's law is no longer valid.

Ferromagnetism

Ferromagnetic substances are those which have strong tendency to move from a region of weak magnetic field to strong magnetic field . Ferromagnetic substances gets strongly magnetised when placed in an external magnetic field. They get strongly attracted to a magnet.In a ferromagnetic material the field lines are highly concentrated.

In non-uniform magnetic field, the sample tends to move towards the region of high field.

5: The value of the susceptibility is positive, but very large **(χ >>1)**

6: Its relative permeability is positive ,very much greater than1**(μ_r>> 1)**

The relative magnetic permeability is >1000 .

7: Its permeability is much grater than absolute permeability,**($\mu >> \mu_0$)**

8: There are a number of elements, which are ferromagnetic: iron, cobalt, nickel, gadolinium,etc.

The explanation for ferromagnetism-

The individual atoms in a ferromagnetic material possess a dipole moment . They interact with one another in such a way that they align spontaneously themselves in a common direction over a macroscopic volume called domain. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about 10^{11} atoms. As shown in fig (a) the magnetisation varies randomly from domain to domain and there is no bulk magnetisation.

When we apply an external magnetic field B_0, the domains orient themselves in the direction of B_0 and simultaneously the domain oriented in the direction of B_0 grow in size. As seen in figure (b) ,the domains have aligned and amalgamated to form a single 'giant' domain.

Hard ferromagnetic materials When the external field is removed ,in some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets. Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle.

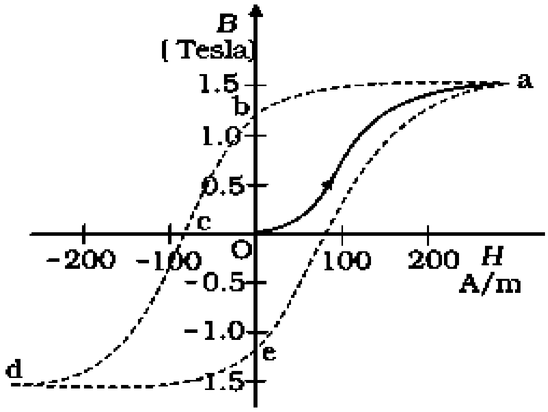
Soft ferromagnetic materials There is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough, such materials are called soft ferromagnetic materials.

Dependence of ferromagnetic property on temperature. At high enough temperature, a ferromagnet becomes a paramagnet. The domain structure disintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the Curie temperature T_c.

The susceptibility above the Curie temperature, i.e., in the paramagnetic phase is described by,

$$\chi = \frac{C}{T - T_c} \quad (T > T_c)$$

HYSTERESIS :-



The relation between B and H in ferromagnetic materials is complex. Figure below shows the behaviour of the material during one cycle of magnetisation Let us consider an unmagnetised ,placed inside a solenoid . When the current through the solenoid increases , the magnetic field B in the material rises and saturates as depicted in the curve Oa. This behaviour represents the alignment and merger of domains until no further enhancement is possible.

If we decrease H and reduce it to zero , i.e. at $H = 0, B \neq 0$. This is represented by the curve ab. The value of B at $H = 0$ is called **retentivity or remanence.**

The domains are not completely randomised even though the external driving field has been removed. When the current in the solenoid is reversed and slowly increased, certain domains are flipped until the net field inside stands nullified. This is represented by the curve bc. The value of H at c is called **coercivity**.

As the reversed current is increased in magnitude, saturation is obtained as depicted by the curve dc .Then the current is reduced ,shown by the curve de & reversed as shown by the curve ea .The cycle repeats itself.

Note that the curve Oa does not retrace itself as H is reduced. For a given value of H, B is not unique but depends on previous history of the sample. This phenomenon is called hysterisis. The word hysterisis means lagging behind .

PERMANENT MAGNETS AND ELECTROMAGNETS

Substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets.

Permanent magnets can be made in following ways:-.

1: By hammering an iron rod repeatedly ,holding in north-south direction

2: Stroking a steel rod with one end of a bar magnet a large number of times, in the same sense .

3: By a ferromagnetic rod in a solenoid and passing a current through the solenoid , so that the magnetic field of the solenoid magnetises the rod.

The hysteresis curve allows us to select suitable materials for permanent magnets.

1-The material should have high retentivity so that the magnet is strong and high coercivity so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations or minor mechanical damage.

2-Further, the material should have a high permeability.

Steel is one-favoured choice for permanent magnet . It has a slightly smaller retentivity than soft iron but this is outweighed by the much smaller coercivity of soft iron.

Other suitable materials for permanent magnets are alnico, cobalt steel and ticonal.

Core of electromagnets are made of ferromagnetic materials which have high permeability and low retentivity.

Soft iron is a suitable material for electromagnets. On placing a soft iron rod in a solenoid and passing a current, we increase the magnetism of the solenoid by a thousand fold. When we switch off the solenoid current, the magnetism is effectively switched off since the soft iron core has a low retentivity.

In certain applications, the material goes through an ac cycle of magnetisation for a long period. This is the case in transformer cores and telephone diaphragms. **The hysteresis curve of such materials must be narrow**. The energy dissipated and the heating will consequently be small. The material must have a high resistivity to lower eddy current losses.

Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery, and bulk quantities of iron and steel.