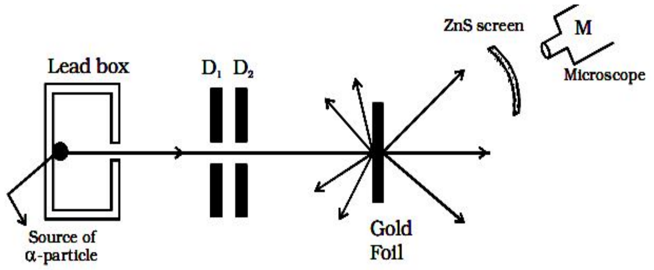
Rutherford's α - particle scattering experiment

Rutherford studied the scattering of the α - particles by a thin gold foil in order to investigate the structure of the atom.

Experimental arrangement

A fine pencil of α–particles was obtained from a radioactive material like radium or radon by placing it in a lead box with narrow opening as shown below



The α-particles are emitted from the source in all possible directions, but only a narrow beam emerges from the lead box. The remaining α-particles are absorbed in the lead box itself. After passing through the diaphragms D_1 and D_2, a narrow beam of α-particles incident on a thin gold foil, are scattered through different angles. The scattered α-particles strike a fluorescent screen coated with zinc sulphide. When the α-particles strike the screen, tiny flashes of light are produced. The observations can be made with the help of a low power microscope .

Rutherford atom model

(i) Atom may be regarded as a sphere of diameter 10^{-10}m, with whole of the positive charge and almost the entire mass of the atom is concentrated in a small central core called nucleus having diameter of about 10^{-14}m .

(ii) The electrons are revolving around the nucleus in circular orbits, so that the centripetal force is provided by the electrostatic force of attraction between the electron and the nucleus.

(iii) As the atom is electrically neutral, the total positive charge of the nucleus is equal to the total negative charge of the electrons in it.

Drawbacks

(i) The electron in the circular orbit experiences a centripetal acceleration. According to electromagnetic theory, an accelerated electric charge must radiate energy in the form of electromagnetic waves. Therefore, if the accelerated electron lose energy by radiation, the energy of the electron continuously decreases and it must spiral down into the nucleus. Thus, the atom cannot be stable. But, it is well known that most of the atoms are stable.

(ii) According to classical electromagnetic theory, the accelerating electron must radiate energy continuously . This will result in a continuous spectrum with all possible wavelengths. But experiments reveal only line spectra of fixed wavelength from atoms.

Distance of closest approach

An α particle directed towards the centre of the nucleus will move close upto a distance r_o, where its kinetic energy will appear as electrostatic potential energy. After this, the α particle begins to retrace its path. This distance r_o is known as the distance of the closest approach.

Let m and v be the mass and velocity of the α particle directed towards the centre of the nucleus. Then, the kinetic energy of the particle

$E_k = \frac{1}{2} mv^2$ …(1)

Since, charge of an α-particle is 2e and that of the nucleus of the atom is Ze, the electrostatic potential energy of the α particle, when at a distance r_o from the centre of the nucleus is given by,

$E_p = (1/4\pi\varepsilon_o)(2e)(Ze)/r_0$ …(2)

On reaching the distance of the closest approach ro, the kinetic energy of the α particle appears as its potential energy.

$$\therefore \quad E_p = E_k$$

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{(2e)(Ze)}{r_o} = \frac{1}{2}mv^2$$

$$\text{(or)} \quad r_o = \frac{1}{4\pi\varepsilon_o}\cdot\frac{4Ze^2}{mv^2} \quad \text{...(3)}$$

Bohr atom model

(i) An electrons can revolve round the nucleus only in those permissible orbits , called stationary orbits , in which the electron does not radiate any energy and the angular momentum of the electron is an integral multiple of h/2π

$$L = mvr = nh/2\pi$$

where h is Planck's constant =6.626×10^{-34} Js , n= 1,2,3 … is called principal quantum number and m and v are the mass and velocity of the electron .

(ii) An atom radiates energy, only when an electron jumps from a stationary orbit of higher energy to an orbit of lower energy.

If the electron jumps from an orbit of energy E_2 to an orbit of energy E_1, a photon of energy $h\nu = E_2 - E_1$ is emitted.

Radius of the nth orbit (r_n)

Let us consider an electron revolve around the nucleus of an atom in the nth orbit of radius r_n ,whose nucleus has a positive charge Ze . .

Since necessary centripetal force is provided by the electrostatic force of attraction between the nucleus and the electron ,

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{Ze^2}{r_n^2} = \frac{mv_n^2}{r_n}$$

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{Ze^2}{r_n} = mv_n^2 \quad \text{...(1)}$$

By Bohr's first postulate,

$$L = mv_nr_n = \frac{nh}{2\pi}$$

$$mv_n^2 = \frac{n^2h^2}{4\pi^2 m r_n^2} \quad \text{...(2)}$$

From equations (1) and (2)

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{Ze^2}{r_n} = \frac{n^2h^2}{4\pi^2 m r_n^2}$$

$$\boxed{r_n = \frac{\varepsilon_o n^2 h^2}{Ze^2 \pi m}} \quad \ldots(3)$$

From equation (3), it is seen that the radius of the nth orbit is proportional to the square of the principal quantum number. Therefore, the radii of the orbits are in the ratio 1 : 4 : 9....

For hydrogen atom, Z = 1

$$r_n = \frac{n^2h^2\varepsilon_o}{\pi m e^2} \quad \ldots\ldots(4)$$

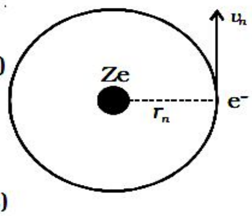
$r_n = n^2 \times 0.53$ Å If n = 1, $r_1 = 0.53$Å This is called Bohr radius.

Energy of an electron in the n^{th} orbit (E_n)

The total energy of the electron is the sum of its potential energy and kinetic energy in its orbit

The potential energy of the electron in the n^{th} orbit is given by,

$$E_p = \frac{(Ze)(-e)}{4\pi\varepsilon_o r_n} = \frac{-Ze^2}{4\pi\varepsilon_o r_n} \quad \ldots(1)$$



The kinetic energy of the electron in the n^{th} orbit is,

$$E_k = \frac{1}{2}mv_n^2 \quad \ldots(2)$$

Since necessary centripetal force is provided by the electrostatic force of attraction between the nucleus and the electron ,

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{Ze^2}{r_n^2} = \frac{mv_n^2}{r_n} \Rightarrow mv_n^2 = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Ze^2}{r_n^2} \quad \ldots3$$

Substuting (3) in (2)

$$E_k = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_o}\ \frac{Ze^2}{r_n}\right] = \frac{Ze^2}{8\pi\varepsilon_o r_n} \quad \ldots(4)$$

The total energy of an electron in its n^{th} orbit is,

$$E_n = E_p + E_k = \frac{-Ze^2}{4\pi\varepsilon_o r_n} + \frac{Ze^2}{8\pi\varepsilon_o r_n}$$

$$\boxed{E_n = \frac{-Ze^2}{8\pi\varepsilon_o r_n}} \quad \ldots(5)$$

Substituting the value of r_n, in (5),

$$E_n = \frac{-Z^2me^4}{8\varepsilon_o^2 n^2 h^2} \quad \ldots(6)$$

For hydrogen atom, Z = 1

$$\therefore \boxed{E_n = \frac{-me^4}{8\varepsilon_o^2 n^2 h^2}}$$

Substituting the known values

$$\boxed{E_n = \frac{-13.6}{n^2}eV} \quad \ldots(7)$$

As there is a negative sign in equation (7), it is seen that the energy of the electron in its orbit increases as n increases.

Frequency of spectral line

According to Bohr's second postulate, when an electron jumps from an outer orbit of quantum number n_2 to an inner orbit of quantum number n_1, the frequency of the photon emitted is given by,

$$\nu = \frac{E_{n_2} - E_{n_1}}{h} \quad \ldots(1)$$

Using the expression for energy

$$\nu = \frac{Z^2me^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \ldots(2)$$

The wave number $\bar{\nu}$ of a radiation is defined as number of waves per unit length. It is equal to reciprocal of the wavelength

$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}$ where c is the velocity of light

$\therefore$ From equation (2) ,

$$\bar{\nu} = \frac{Z^2me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \ldots(3)$$

For hydrogen atom, Z=1

$$\bar{\nu} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \ldots(4)$$

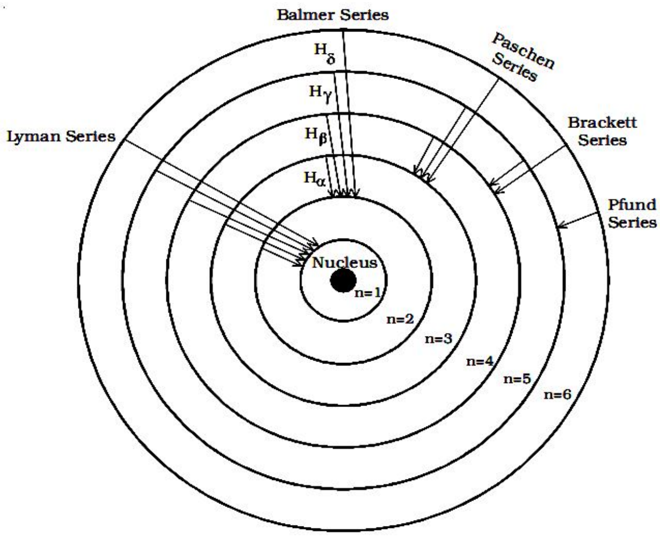
where R is a constant called Rydberg's constant

$$\therefore \boxed{R = \frac{me^4}{8\varepsilon_o^2 ch^3}} \quad \ldots(5)$$

Substituting the known values, we get

$R = 1.094 \times 10^7 m^{-1}$

Spectral series of hydrogen atom



Whenever an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference in energies of the two levels is emitted as a radiation of particular wavelength. It is called a spectral line. As the wavelength of the spectral line depends upon the two orbits (energy levels) between which the transition of electron takes place, various spectral lines are obtained. The different wavelengths constitute spectral series which are the characteristic of the atoms emitting them. The following are the spectral series of hydrogen atom.

(i) Lyman series When the electron jumps from any of the outer orbits to the first orbit, the spectral lines emitted are in the ultraviolet region of the spectrum and they are said to form a series called Lyman series .

Here, $n_1 = 1$, $n_2 = 2,3,4$ …

Wave number of the Lyman series is given by,

$$\bar{\nu} = R\left(1 - \frac{1}{n_2^2}\right)$$

(ii) Balmer series When the electron jumps from any of the outer orbits to the second orbit, we get a spectral series called the Balmer series. All the lines of this series in hydrogen have their wavelength in the visible region.

Here $n_1= 2$, $n_2 = 3,4,5$ …

The wave number of the Balmer series is,

$$\bar{\nu} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{4} - \frac{1}{n_2^2}\right)$$

The first line in this series ($n_2 = 3$), is called the H_α-line, the second ($n_2=4$),the H_β-line and so on.

(iii) Paschen series This series consists of all wavelengths which are emitted when the electron jumps from outer most orbits to the third orbit.

Here $n_2= 4,5,6$ … and $n_1 = 3$.

This series is in the infrared region with the wave number given by

$$\bar{\nu} = R\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{9} - \frac{1}{n_2^2}\right)$$

(iv) Brackett series The series obtained by the transition of the electron from $n_2 = 5, 6$...to $n_1= 4$ is called Brackett series. The wavelengths of these lines are in the infrared region.

The wave number is,
$$\bar{\nu} = R\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{16} - \frac{1}{n_2^2}\right)$$

(v) Pfund series The lines of the series are obtained when the electron jumps from any state $n_2 = 6, 7$... to $n_1=5$. This series also lies in the infrared region.

The wave number is,
$$\bar{\nu} = R\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{25} - \frac{1}{n_2^2}\right)$$

Energy level diagram

The energy of the electron in the nth orbit of the hydrogen atom is given by,

$$E_n = -13.6 / n^2 \text{ eV}$$

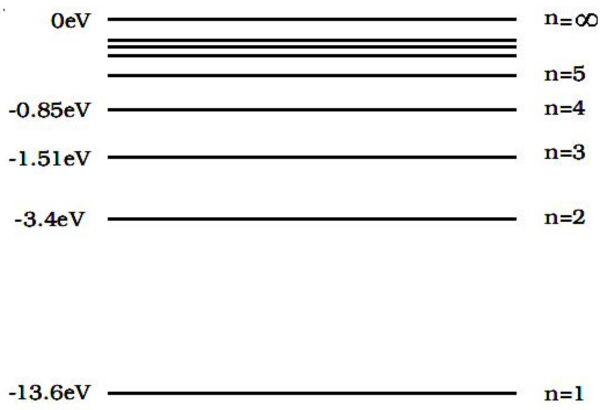
Energy associated with the first orbit of the hydrogen atom is,

$$E_1 = - 13.6 / 1^2 = -13.6 \text{ eV}$$

It is called ground state energy of the hydrogen atom.

Energy associated with the second orbit is given by,

$E_2 = -13.6 / 2^2 = -3.4$ eV



It is called energy of first excited state of the hydrogen atom. The energy of second, third, fourth, … excited states of the hydrogen atom are,

$E_3 = -1.51$ eV,

$E_4 = -0.85$ eV,

$E_5 = -0.54$eV ...

when $n = \infty$, $E_\infty = -13.6 / \infty^2 = 0$

Therefore, it is seen from the above values, that, the energy associated with a state becomes less negative and approaches closer and closer to the maximum value zero corresponding to $n = \infty$.

Taking these energies on a linear scale, horizontal lines are drawn which represent energy levels of the hydrogen atom .. This diagram is known as energy level diagram.

Limitations of Bohr's atomic model .

(i)The Bohr model is applicable to hydrogenic atoms. It cannot be extended even to mere two electron atoms such as helium. The analysis of atoms with more than one electron was attempted on the lines of Bohr's model for hydrogenic atoms but did not meet with any success.Difficulty lies in the fact that each electron interacts not only with the positively charged nucleus but also with all other electrons.The formulation of Bohr model involves electrical force between positively charged nucleus and electron. It does not include the electrical forces between electrons which necessarily appear in multi-electron atoms.

(ii)While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, the model is unable to explain the relative intensities of the frequencies in the spectrum. In emission spectrum of hydrogen, some of the visible frequencies have weak intensity, others strong. Experimental observations depict that some transitions are more favoured than others. Bohr's model is unable to account for the intensity variations.