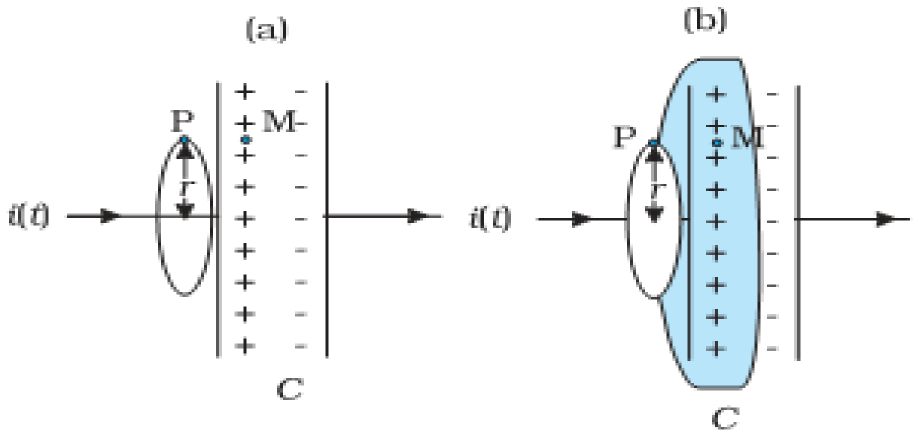
NCERT-XII / Unit- 08 / EM Wave

DISPLACEMENT CURRENT

The existence of radio waves, gamma rays and visible light, as well as all other forms of electromagnetic waves can be explained on the basis of production of a magnetic field by changing an electric field.

Give the origin of Displacement current . **Or,**

Explain how a changing electric field gives rise to a magnetic field ?



During the process of charging of a capacitor , the magnetic field at a point P, in a region outside the parallel plate capacitor , can be calculated by considering a plane circular loop of radius r , around the current-carrying wire , as shown in Fig.(a)

So applying Ampere's circuital law $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i(t)$,

we have, B (2πr) = μ_0i (t) = > $\mathbf{B = \mu_0 i / 2\pi r}$… (2)

Now, let us consider a different surface, with same boundary as shown in fig.(b) which has its bottom between the capacitor plates .

Again applying Ampere's circuital law to such surfaces with the same perimeter, we find that $\oint \mathbf{B}\cdot d\mathbf{l} = 0$ => B = 0 , since no current passes through the surface of fig. (b) .

So we have a contradiction of having a magnetic field at a point P; and calculated another way, no magnetic field at P .

So the Ampere's circuital law, must be missing a term, such that one gets the same magnetic field at point P, no matter what surface is used.

Since the same electric field of magnitude E = σ / ε_0 is passing perpendicularly through the surface S and between the plates of the capacitor of area A, where σ = **Q/A**, using Gauss's law, $\Phi_E = |\mathbf{E}| A = \frac{1}{\varepsilon_0}\frac{Q}{A} A = \frac{Q}{\varepsilon_0}$ .... (3)

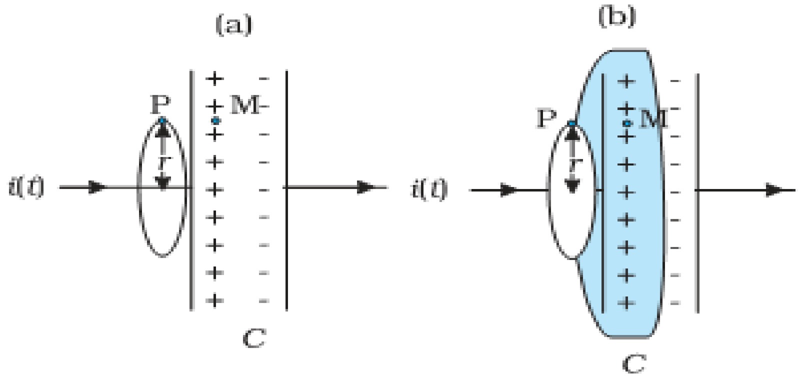
Differentiating Eq. (3), we have

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

$$\Rightarrow \varepsilon_0\left(\frac{d\Phi_E}{dt}\right) = \frac{dQ}{dt} = i_d \quad \text{........................ (4)}$$

This is the missing term in Ampere's circuital law and known as **displacement current** , which is ε_0 times the rate of change of electric flux through the same surface.

Deduce Ampere's Maxwell law



During the process of charging of a capacitor , the magnetic field at a point P, in a region outside the parallel plate capacitor , can be calculated by applying Ampere's circuital law , as **B = μ_0i /2πr** by considering a plane circular loop of radius r , around the current-carrying wire , as shown in Fig.(a) .

Now, let us consider a different surface, with same boundary as shown in fig.(b) which has its bottom between the capacitor plates . Since no current passes through the surface of fig. (b) . Applying Ampere's circuital law to such surfaces with the same perimeter, we find that B = 0 , at the same point P .

So Ampere's circuital law, must be missing a term , as we have a contradiction of having a magnetic field at a point P; and calculated another way, no magnetic field at P .

Since the magnitude of electric field between the plates of the capacitor of area A is $E = \sigma / \varepsilon_0$, passing perpendicularly through the surface S , where $\sigma = Q/A$,

using Gauss's law, $$\Phi_E = |\mathbf{E}| A = \frac{1}{\varepsilon_0}\frac{Q}{A}A = \frac{Q}{\varepsilon_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Differentiating Eq. (3),

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt} \Rightarrow \varepsilon_0\left(\frac{d\Phi_E}{dt}\right) = \frac{dQ}{dt} = i_d \quad \ldots.. (4)$$

This is the missing term in Ampere's circuital law and known as **displacement current** , The source of a magnetic field is not just the conduction electric current due to flowing charges, but also the time rate of change of electric field. So, the total current i is the sum of the conduction current denoted by i_c, and the displacement current denoted by i_d ($= \varepsilon_0$ ($d\Phi_E/dt$)).

So we have $$i = i_c + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt}$$

Outside the capacitor plates, we have only conduction current $i_c = i$,

and no displacement current, i.e., $i_d = 0$.

On the other hand, inside the capacitor, there is no conduction current, i.e., $i_c = 0$,

and there is only displacement current, so that $i_d = i$.

The generalised Ampere's circuital law ca be stated as **"the total current passing through any surface of which the closed loop is the perimeter is the sum of the conduction current and the displacement current"**.

The generalised law is

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

This is known as Ampere-Maxwell law.

The displacement current has the same physical effects as the conduction current.
In steady electric fields in a conducting wire, the displacement current is zero since the electric field E does not change with time.
In a charging capacitor , both conduction and displacement currents present in different regions of space.
They may be present in the same region of space, if the medium is not perfectly conducting or perfectly insulating .
There may be large regions of space where there is no conduction current, but there is only a displacement current due to time-varying electric fields.
In such a region, we expect a magnetic field, though there is no conduction current source nearby .

Consequences of displacement current.

The laws of electricity and magnetism are now more symmetrical*. Faraday's law of induction states that there is an induced emf equal to the rate of change of magnetic flux. Now, since the emf between two points 1 and 2 is the work done per unit charge in taking it from 1 to 2, the existence of an emf implies the existence of an electric field.
So, Faraday's law of electromagnetic induction can be stated by saying that a magnetic field, changing with time, gives rise to an electric field.
Then, the fact that an electric field changing with time gives rise to a magnetic field, is the symmetrical counterpart, and is a consequence of the displacement current being a source of a magnetic field.

Thus, time- dependent electric and magnetic fields give rise to each other .

Faraday's law of electromagnetic induction and Ampere-Maxwell law give a quantitative expression of this statement, with the current being the total current, as $i = i_c + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt}$

One very important consequence of this symmetry is the existence of electromagnetic waves.

MAXWELL'S EQUATIONS

1. $\oint \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$ (Gauss's Law for electricity)
2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ (Gauss's Law for magnetism)
3. $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\Phi_B}{dt}$ (Faraday's Law)
4. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ (Ampere – Maxwell Law)